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ABSTRACT

This concept module on momentum is one of thirteen modules that provide a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, postsecondary technical schools. Each of the thirteen concept modules discusses a single physics concept and how it is applied to each energy system. This module explains the mathematical relationships involved with both linear and angular momentum. Elastic and inelastic collisions are considered, as well as the law of conservation of momentum. Included with the concept module are five application modules that discuss a specific application of a single concept in a single energy system, and contain a student laboratory exercise. Laboratory modules included are Angular Momentum of a Flywheel, Impulse and Momentum, Conservation of Momentum in Collisions, Gyroscopes, and Rocket Engines. (LRA)

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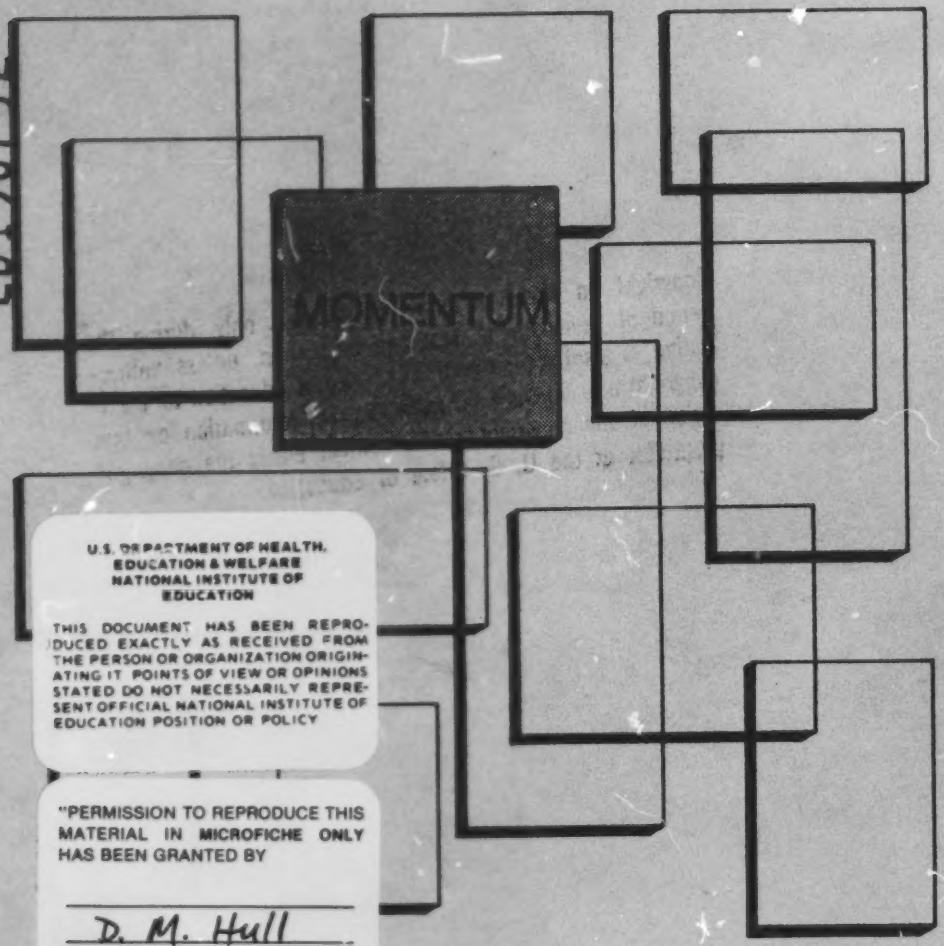
UTC

UNIFIED TECHNICAL CONCEPTS

CONCEPT MODULE 4-0



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INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet

specific needs of students in a specific technology.

STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "forcelike quantities" in the following energy "systems":

- In mechanical systems <u>force</u> and <u>torque</u> cause motion of mass.
- In fluidal systems <u>pressure difference</u> causes motion of fluids.
- In electrical systems <u>potential difference</u> causes motion of mass.
- In thermal systems <u>temperature difference</u> causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

COMPONENTS OF UTC

UTC incorporates three types of modules.

CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

1-0 Force.
2-0 Work.
3-0 Rate.
4-0 Momentum.
5-0 Resistance.
6-0 Power.
7-0 Potential and Kinetic Energy.
8-0 Force Transformers.
9-0 Energy Convertors.
10-0 Transducers.
11-0 Vibrations and Waves.
12-0 Time Constants.
13-0 Radiation.

APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires. In the laboratory, students calculate and measure experimentally the resistance of various wires.

SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

S1 Formula Interpretation.
S2 International System of Units.
S3 Logarithms and Exponents.
S4 Angles and Triangles.
S5 Vectors and Scalars.
S6 Dimensional Analysis.
S7 Reading and Drawing Graphs.
S8 Precision, Accuracy, and Measurement.

STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

INTRODUCTION

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

PREREQUISITES

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

OBJECTIVES

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

DISCUSSION

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

LABORATORY (called DEMONSTRATIONS in the Concept Modules)

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

SELF CHECK

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

REFERENCES

This list of sources directs students to additional information on topics discussed in the module.

TABLE OF CONTENTS

4-0 Momentum
4M1 Angular Momentum of a Flywheel
4M2 Impulse and Momentum
4M3 Conservation of a Momentum in Collisions
4M4 Gyroscopes
4F1 Rocket Engines

INTRODUCTION

Moving objects possess a very important physical quantity called "momentum." "Linear momentum," a vector quantity, depends upon the mass and the linear velocity of the object. "Angular momentum," also a vector quantity, depends upon the moment of inertia and the angular velocity of a rotating mass.

This module explains the mathematical relationships involved with both linear and angular momentum. Elastic and inelastic collisions are considered, as well as "the law of conservation of momentum."

The law of conservation of momentum is an important unifying principle in physics. Whenever momentum is transferred from one object to another, or from one system to another, total momentum always is conserved. The concept of momentum finds the greatest practical application in mechanical and fluid systems.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms; state the appropriate units in the mk/s system (SI) and the cg/s system; and give the equation for each:

  Linear momentum.
  Angular momentum.
  Impulse.
  Angular impulse.
  Moment of inertia.

- Given two of the following quantities, determine the third:

  Mass of an object.
  Velocity of the object.
  Momentum of the object.

- Given two of the following quantities, determine the third:

  Moment of inertia of an object.
  Angular velocity of the object.
  Angular momentum of the object.

- Given all the following quantities except one describing a linear collision, determine the unknown quantity:

  Mass of first object.
  Initial velocity of first object.
  Final velocity of first object.
  Mass of second object.
  Initial velocity of second object.
  Final velocity of second object.

- Explain the following concepts in a short paragraph each:

  Conservation of linear momentum.
  Conservation of angular momentum.

- Use a given equation to calculate the force produced on one blade of a reaction turbine, given the velocity of fluid and the mass of fluid per unit time striking the blade.

DISCUSSION

A large ocean liner has a tremendous mass. When such a liner approaches a dock (Figure 1), the crew reduces the ship's velocity to a very low value. As the enormous ship contacts the dock, a hugh force must be absorbed by the dock in order to stop the ship's motion completely. Very massive objects are difficult to stop.

FIG. 1

A bullet fired from a gun at high velocity has a tremendous impact on a target, although the mass of the bullet may be only a few grams. The huge force exerted by the bullet upon the target can be seen by the destructive power of even a small-caliber bullet fired at a glass bottle (Figure 2). If the bottle is filled with water, the velocity of the bullet is reduced more upon impact; and the destruction of the target is even greater. Rapidly-moving objects are difficult to stop.

FIG. 2

These examples illustrate the concept of momentum. Momentum is an important physical quantity related to the motion of objects. Either a small object moving with a high velocity, or a large object moving with a low velocity, possesses relatively large amounts of momentum and exerts large forces on other objects when stopped. Both mass and velocity are necessary in determining the momentum of a body.

Considerations of momentum for moving objects is divided into two classifications:

(1) Linear momentum if the object is moving in a straight line.
(2) Angular momentum if the object is rotating.

LINEAR MOMENTUM

Linear momentum of a single object of mass moving with velocity v is defined by Equation 1.

$$\vec{p} = m\vec{v} \qquad (1)$$

where
p = linear momentum.
m = mass.
v = velocity.

(The arrow drawn over p and v simply indicates that they are vectors.)

Since velocity is a vector quantity (one that has both magnitude and direction), momentum also must be a vector quantity if Equation 1 is to hold true. The direction of the momentum vector is the same

as that of the velocity vector, but its magnitude and units are different from that of the velocity vector. The vector nature of momentum will be considered more thoroughly later in this module. As indicated in Equation 1, arrows always are drawn over vector quantities when the direction, as well as the magnitude, is important in the problem. In momentum problems, both magnitude and direction are important.

Units for momentum are consistent with Equation 1. If mk/s units are used for mass (kg) and velocity (m/s), then the momentum units obtained would be (kg-m/s). However, if the cg/s system is used, mass in grams times velocity in cm/s yields momentum in units of g-cm/s. Examples A and B illustrate the calculation of momentum for two greatly-different masses.

EXAMPLE A: LINEAR MOMENTUM OF LARGE MASS

Given: An automobile whose mass is 2500 kg is moving with a velocity of 70 km/h. (Assume that the car is moving in a straight line.)

Find: Its momentum.

Solution: The momentum of the automobile will be in the same direction as its velocity.

$$\vec{p} = m\vec{v}$$

$$= (2500 \text{ kg})\left(\frac{70{,}000 \text{ m}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$$

$$\vec{p} = 48{,}600 \text{ kg-m/s}$$

EXAMPLE B: LINEAR MOMENTUM OF SMALL MASS

Given: An electron whose mass is 9.1×10^{-28} grams is moving in an electric wire at an effective velocity of approximately 0.1 cm/s.

Find: Its momentum.

Solution: $\vec{p} = m\vec{v}$

$$= (9.1 \times 10^{-23} \text{ g})(1 \times 10^{-1} \text{ cm/s})$$

$\vec{p} = 9.1 \times 10^{-29}$ g-m/s (a small momentum!)

IMPULSE AND MOMENTUM CHANGE

When force is applied to a moving object, velocity change is produced. The velocity either may increase or decrease, depending upon the direction in which the force is applied to the object. The change in velocity requires a corresponding change in momentum of the object, because momentum depends upon the velocity, as indicated in Equation 1. The amount of change in momentum is equal to the product of the force and the elapsed time during which the force acts. This relationship is expressed by Equation 2.

$$\Delta p = F\Delta t \qquad (2)$$

where

Δp = change in momentum.
F = force.
Δt = elapsed time during which the force acts.

The term applied to the product of force and time is "impulse."

The concept of impulse and momentum change is understood better when analyzed with the following short derivation:

$$F = ma \text{ (Newton's second law)}$$

$$F = m\frac{\Delta v}{\Delta t} \text{ (since } a = \frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time elapsed}}\text{)}$$

$$F\Delta t = m\frac{\Delta v}{\Delta t}\Delta t \text{ (multiplying both sides by } \Delta t\text{)}$$

$$F\Delta t = m\Delta v$$

Impulse = change in momentum

From this derivation, impulse is equivalent to the change in momentum produced when a force is applied to an object. Consider the case of a baseball being struck by a bat. The baseball is given its initial momentum and velocity by the pitcher. The direction of these quantities is, of course, toward the batter. As the bat contacts the ball, an enormous force is applied to the ball for the short time during which the two objects are in contact. This force is large enough to cause a considerable change in the momentum and velocity of the ball. The direction of the ball is changed, as well as is its speed. The impulse, or the product of the force applied and the time during which the bat and the ball were in contact, is equal to the change in momentum of the ball, provided the vector nature of momentum is considered.

FIG. 3

In most "real-life" examples of impulse and changes in momentum, the amount of force varies during the time of contact; however, "average" force usually can be determined in order to complete the necessary calculations. For example, the graph in Figure 3 represents the variation of the force applied to the ball during the contact time.

The shaded area under the curve in Figure 3 is equal to the impulse. The irregular shape of this curve makes an exact calculation of impulse very difficult. Instead, an average constant force can be found, as in Figure 4. If the average force, represented by the straight line in Figure 4, were applied continually to the ball throughout the same contact time, the same impulse would be produced. This is true, because the shaded areas under the curves in Figures 3 and 4 (i.e., impulse) are equal. The shaded areas are made equal, of course, by use of the correct average force. Average force is determined in this manner.

FIG. 4

CONSERVATION OF MOMENTUM

The total momentum of an isolated system does not change; it is conserved. An isolated system is one in which there are no external, unbalanced forces acting upon the system. For example, the forward momentum of a baseball does not change noticeably between the time that it leaves the pitcher's hand until it is struck by the bat, because the only force acting upon it in the direction of motion is resistance, which is relatively small. A large momentum is given to a golf ball on a drive. Because the golf ball is in the air for a longer time than the baseball, much of its momentum gradually is transferred to the molecules in the air and apparently lost. However, the momentum gained by the molecules in the air is equal to the momentum lost by the ball. In this case, the isolated system must include both the golf ball and the surrounding air. Conservation of momentum applied to all isolated systems, whether the systems be colliding atoms, billiard balls, automobiles, or galaxies of stars.

FIG. 5

Consider an isolated system of boat and boater. The boater steps from the boat to a dock, as illustrated in Figure 5.

As the boater steps forward from the boat, a force is applied to the boat, as well as to the boater. The boater pushes back on the boat, but the boat pushes forward on the boater with an equal force. These forces cause the boater of mass m_1 to move forward with velocity v_1 at the same time that the boat of mass m_2 moves backward with velocity v_2. Because momentum is conserved, the product m_1v_1 (the boater's momentum) is equal to the product m_2 (the momentum of the boat): the initial momentum of boater and boat was zero before the boater stepped from the boat to the dock. After the boater steps from the boat, the total momentum must still be zero, since momentum is conserved. But the boater's

momentum when stepping from the boat is $m_1\vec{v}_1$ while the boat's momentum is $m_2\vec{v}_2$. The sum of $m_1\vec{v}_1 + m_2\vec{v}_2$ must remain zero. Thus, $m_1\vec{v}_1 = -m_2\vec{v}_2$. The minus sign indicates that boater and boat move away from each other in opposite directions, each with the same numerical value of momentum. The total momentum of boater and boat in this system is zero before, during, and after the boater steps from boat to dock; and momentum is conserved.

An important and interesting observation to be made from the above example is that a boat with a much larger mass will "recoil" with a lower velocity. Thus, the velocity of the boat backward will be much greater for a lightweight canoe than for a heavy runabout. To step from a heavy boat is much easier than from a lightweight boat. Conservation of momentum in this example can be summarized by Equation 3.

$$m_1\vec{v}_1 = -m_2\vec{v}_2 \qquad (3)$$

where

m_1 = mass of boater.
m_2 = mass of boat.
v_1 = velocity of boater.
v_2 = velocity of boat.

An important application of the conservation of momentum occurs in the firing of a rifle. When the rifle fires, the expanding gases push upon both the bullet and the rifle equally. The force produces a much larger velocity upon the small bullet than upon the massive rifle. As a result, the bullet leaves the barrel of the rifle with a very large velocity. The massive rifle, however, recoils with a much lower velocity. A relation similar to Equation 3 also applies to this case, since momentum is conserved. Equation 4 summarizes the results, and Example C shows how recoil velocity for a rifle can be calculated from Equation 4.

$$m_2\vec{v}_2 = -m_1\vec{v}_1 \qquad (4)$$

where

m_1 = mass of bullet.
m_2 = mass of rifle.
v_1 = velocity of bullet.
v_2 = velocity (recoil) of rifle.

EXAMPLE C: CONSERVATION OF MOMENTUM AND RIFLE RECOIL VELOCITY

Given: A bullet of mass 5 g is fired from a rifle whose mass is 6 kg. The velocity of the bullet is 675 m/s when it leaves the rifle barrel.

Find: The recoil velocity of the rifle.

Solution: Both rifle and bullet considered separately — but as part of the same isolated system — are at rest before firing. Thus, the initial momentum of bullet and rifle is zero. Since momentum is conserved, the final momentum of bullet and rifle also is zero. Final momentum of the system (bullet and rifle) can be written as follows:

$$m_2v_{2_{rifle}} + m_1v_{1_{bullet}} = 0$$

or

$$m_2v_{2_{rifle}} = -m_1v_{1_{bullet}}$$

The negative sign simply is a reminder that the recoil velocity of the rifle is in a direction opposite to the velocity of the bullet. The numerical values of $m_2v_{2_{rifle}}$ and $m_1v_{1_{bullet}}$ are equal, so we can ignore the negative sign and write:

$$m_2v_2 = m_1v_1$$

The recoil velocity of the rifle is given by v_2, which is —

$$v_2 = \frac{m_1v_1}{m_2}$$

Substitution of the numerical values yields —

$$v_2 = \frac{5\ g \times 675\ \frac{m}{s}}{6000\ g}$$

$$v_2 = 0.56\ m/s$$

The rifle kicks back with a recoil velocity of 0.56 m/s.

Another common application of the law of conservation of momentum involves the collision of two or more objects. Each object may have momentum before the collision, and each may have momentum after the collision. When Equations 3 or 4 are generalized to account for the collision of two or more objects, Equation 5 is obtained.

$$(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 \ldots) = (\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots) \qquad (5)$$

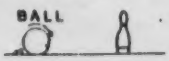
where

$\vec{P}$ = momentum of object(s) before collision.
$\vec{p}$ = momentum of object(s) after collision.

Equation 5 is used to solve a problem in Example D, which considers a bowling ball colliding with a stationary pin in such a manner that all motion takes place in a straight line.

EXAMPLE D: CONSERVATION OF MOMENTUM - COLLISION WITH A STATIONARY OBJECT

Given: A 7-kg bowling ball moving with a velocity of 2 m/s collides head on with a bowling pin. Immediately after the collision, the 1.0-kg pin moves with a velocity of 6 m/s.



Velocities are in the same line (co-linear).

Find: Velocity of the ball immediately after the collision.

Solution: Equation 5 applies, since momentum must be conserved:

$$(P_{ball} + P_{pin}) = (p'_{ball} + p'_{pin})$$

Before After

P_{ball} = mass ball x velocity before collision.
= (7 kg)(2 m/s) = 14 kg-m/s.

P_{pin} = 0, since the pin has zero velocity before the collision.

p'_{ball} = (7 kg)(unknown velocity, v_2).

p'_{pin} = (1.0 kg)(6 m/s) = 6 kg-m/s.

Thus, use of the equation above yields —

$$(14 \text{ kg-m/s} + 0) = [(7 \text{ kg})(v_2) + 6 \text{ kg-m/s}]$$

Before After

Solve for v_2 —

$$(7 \text{ kg})(v_2) = 8 \text{ kg-m/s}$$

$$v_2 = 1.1 \text{ m/s}$$

The bowling ball thus continues straight ahead after the collision, with a speed of 1.1 m/s. Note: Friction of the ball against the floor and the spin on the ball may alter this result slightly. Friction represents an external force; therefore, this example is not a totally isolated system.

Momentum is conserved in all collisions. When a certain type of collision occurs, in which all the kinetic energy possessed by the objects remains as kinetic energy after the collision, it is called an "elastic collision." When some, or all, the kinetic energy is changed to other forms of energy during the collision, an "inelastic collision" results. (Kinetic energy can be changed to heat energy due to friction or can be used in changing the shapes of the objects on collision.) Of course, the total amount of all forms of energy is conserved in both types of collisions. Perfectly elastic collisions are not common and only occur in collisions between atoms and subatomic particles. Inelastic collisions are very common in all "real world" situations.

ANGULAR MOMENTUM

Angular momentum is very similar to linear momentum. In practice, it often refers to objects rotating about an axis. A massive flywheel rotating at the end of an engine crankshaft has large angular momentum. It is used to smooth the rotation of an engine between the firing of the pistons. Because of its large angular momentum, it tends to keep rotating evenly between firings, avoiding the otherwise jerky motion which would accompany the spaced firings of the pistons.

The mathematical relationship defining angular momentum is similar to the defining equation for linear momentum. Compare Equation 6 with Equation 1.

$$\vec{L} = I\vec{\omega} \qquad (6)$$

where
$\vec{L}$ = angular momentum.
I = moment of inertia.
ω = angular velocity.

In this equation, ω is the angular velocity, defined in Concept Module 3-0, "Rate." Angular velocity is the rate of angular rotation and in this equation must be measured in units of radians/s. The moment of inertia I is a quantity similar to mass in a translational mechanical system. The location of the axis of rotation of the object greatly affects the moment of

inertia. If a small particle of mass, such as a grain of sand, is spinning about an axis a distance r from the grain of sand, the moment of inertia of the sand is equal to mr^2, where m is the mass of the sand particle. For large objects, the moment of inertia varies according to the shape of the objects and the location of the axis rotation. A few examples for moment of inertia of regular bodes are given in Figure 6.

Units for angular momentum can be determined by substitution into Equation 6:

$$L = I\omega \rightarrow (kg\text{-}m^2)\left(\frac{rad}{s}\right) \rightarrow \frac{kg\text{-}m^2}{s}$$

Thus, in the mk/s system, angular momentum has units of $kg\text{-}m^2/s$. In the cg/s system —

$$L = I\omega \rightarrow (g\text{-}cm^2)\left(\frac{rad}{s}\right) \rightarrow \frac{g\text{-}cm^2}{s}$$

Note: As discussed in Module 3-0, the radian is a "dimensionless unit" and can be omitted in expressions of units.

Example E indicates how angular momentum is calculated for a flywheel.

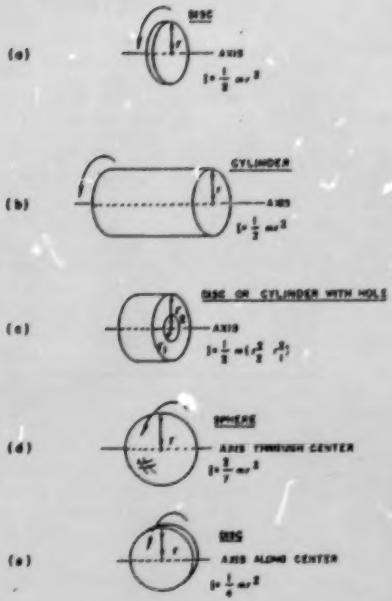
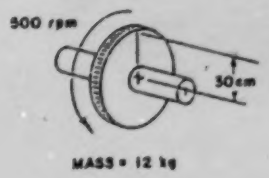


FIG. 6

EXAMPLE E: ANGULAR MOMENTUM OF A FLYWHEEL

Given: A 30-cm (radius) flywheel with a mass of 12 kg is rotating about an axis through its center at 500 r/min (rpm).



Find: Its angular momentum.

Solution: $L = I\omega$

$I = 1/2\ mr^2$, since a flywheel is a disk rotating as shown in Figure 7a.

$$\omega = 500\ \frac{r}{min} \times \frac{1\ min}{60\ s} \times \frac{2\pi\ rad}{r} = 52.4\ \frac{rad}{s}$$

$$L = 1/2\ (12\ kg)(0.3\ m)^2\left(52.4\ \frac{rad}{s}\right)$$

$$L = 28.3\ \frac{kg\text{-}m^2}{s}$$

Recall that linear momentum can be changed when a force is applied for a given time (impulse). Similarly, angular momentum can be changed when a torque is applied for a given time. Angular impulse is a torque (forcelike quantity) applied for a given time, as summarized in Equation 7.

$$\Delta L = \tau \Delta t \qquad (7)$$

where

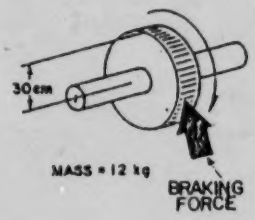
ΔL = change in angular momentum (angular impulse).
τ = torque.
Δt = elapsed time.

Relating this to the flywheel on the engine crankshaft, each time a piston fires, an angular impulse — a torque acting for a short time — acts on the engine shaft to increase its angular momentum. Since the moment of inertia of the crank shaft remains fixed, angular velocity is the quantity that increases. Angular velocity is transformed by the differential to increase the linear speed of the car. In Example F, the amount of angular impulse produced by a certain braking torque when applied to a rotating flywheel is examined.

EXAMPLE F: ANGULAR IMPULSE AND CHANGE IN ANGULAR MOMENTUM

Given: A braking torque of 5 N·m is applied to the flywheel in Example E for 2.5 seconds. Initial angular momentum, as worked out in Example E, is 28.3 $kg\cdot m^2/s$.

Find: (a) The angular impulse produced and (b) the angular momentum of the flywheel after the braking action.



Solution: (a) $\Delta L = \tau \Delta t$

$= (5\ N \cdot m)(2.5\ s)$

$\Delta L = 12.5\ N \cdot m/s$

The units for angular impulse appear to cause trouble at first, because they are not the same as those for angular momentum. A short dimensional analysis is necessary to verify that these units are equivalent.

Substitution:

$$12.5\ \text{Newton}\ (m/s) = 12.5\ \left(\frac{kg \cdot m}{s^2}\right)(m/s)$$

$$= 12.5\ \frac{kg \cdot m^2}{s}$$

(b) Thus, the change in angular momentum of the flywheel is 12.5 $kg \cdot m^2/s$. The angular momentum of the flywheel after the braking action then is this:

$$\text{original value} - \text{change} = \text{final value}$$

$$28.3\ \frac{kg \cdot m^2}{s} - 12.5\ \frac{kg \cdot m^2}{s} = 15.8\ \frac{kg \cdot m^2}{s}$$

The law of the conservation of angular momentum is as important as the laws of conservation of linear momentum and the law of the conservation of energy. It simply states that "the angular momentum of a system remains constant, provided no external torques are applied"; that is, the system is an "isolated" one.

A complete discussion of the mathematical and vectorial analysis of angular momentum is beyond the objectives of this module; however, the results of the conservation of angular momentum in common applications are relevant.

An observation that can be made from Equation 6 (i.e., $L = I\omega$) is that several "hidden variables" are present. Both mass and radius of rotation are included in the moment of inertia term I. These facts are explained further in Examples G and H.

EXAMPLE G: ANGULAR MOMENTUM AND ICE SKATERS

Given: Skaters practicing pirouettes extend their arms to slow down, and hold their arms tight to their body to increase their angular speed.

Find: The reason why.

Solution: A skater rotating must conserve angular momentum, like all other isolated systems. Thus, angular momentum while arms are extended must equal the angular momentum while arms are tucked in. When arms are extended, the effective radius of rotation of the skater becomes larger; thus, the angular velocity must be less for angular momentum to be conserved. In other words, the product $I\omega$ must remain constant; as I increases with a larger radius, ω must decrease. The same principle is used by platform divers and ballet dancers.

EXAMPLE H: MERRY-GO-ROUNDS AND ANGULAR MOMENTUM

Given: A merry-go-round turning with no one pushing will slow down when a heavy person gets on.

Find: The reason why.

Solution: The angular momentum of the merry-go-round before and after the additional person gets on must remain the same. When the heavy person gets on the merry-go-round, the mass of the system increases. This increased mass causes the value for I to increase in the equation $I\omega$; thus, ω must decrease to keep the product unchanged. The product must remain unchanged if angular momentum is to be conserved.

FLUIDAL SYSTEMS

When a stream of fluid hits a dish-shaped blade in the manner shown in Figure 7, the direction of the fluid is reversed.

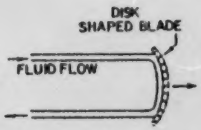


FIG. 7

The momentum of the moving fluid is changed, because its direction has been reversed. This change of momentum means that an impulse has been produced. The amount of force produced by this impulse can be deduced from Equation 2 and is given by Equation 8.

$$F = 2\left(\frac{m}{t}\right)v \qquad (8)$$

where

F = force on blade.
m = mass of fluid.
t = time.
v = velocity of fluid.

FIG. 8

This equation is the principle behind devices known as reaction turbines. The most common example of a reaction turbine is a common lawn sprinkler (Figure 8). The reaction to the change in momentum of the water as its direction is changed causes the arms to spin.

Any hose or pipe that forces a change in direction of a fluid causes a momentum change in the fluid and so must be strong enough to absorb the reaction to this momentum change. The reaction can be seen as an effort by the system to conserve momentum.

SUMMARY

The law of conservation of momentum is itself a unifying concept, because it is applied to any moving object or systems of objects free of external forces affecting the motion. This concept can include objects of all kinds, including moving fluids, moving electrons, or billiard balls. Momentum is used most often in connection with mechanical systems. The concept of momentum also is useful in connection with fluidal systems. The concept of momentum, at the level studied, is not found often to have practical application in the studies of heat or electricity.

LINEAR MOMENTUM

MATERIALS

Air track or air table and accessories (optional).
Two dry ice pucks (Cenco #75865-000 or equivalent).
Dry ice.
Two meter sticks.
Two stopwatches.

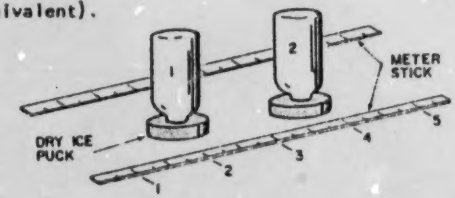


FIG. 9

PROCEDURE

If an air track or air table is available, use it to perform the momentum experiments suggested by the manufacturer or the equipment (optional).

Verify the conservation of linear momentum.

1. Position two identical dry-ice pucks on a smooth surface and between two meter sticks (Figure 9). The only purpose of the meter sticks is to act as tracks to assure linear motion of the pucks.

2. With puck #2 stationary and positioned approximately in center of the track, give puck #1 a slight push. Using a stopwatch, measure time required for puck #1 to travel distance between two points which you determine. From known distance and measured time, the average speed of puck can be calculated by the formula $v = d/t$. Record results in Data Table.

3. As puck #1 collides with puck #2, its momentum is transferred. But since the mass of puck #1 and puck #2 are equal, their speed is a direct measurement of their momentums. To verify the conservation of linear momentum, we must, therefore, show that the speed of puck #2 after the collision is the same as the speed of Puck #1 before the collision. Mathematically, this argument is as follows:

$$P_1 + P_2 = P_1' + P_2'$$

$$mv_1 + mv_2 = mv_1' + mv_2'$$

Since mass is equal for each puck, the following is true:

$$v_1 + v_2 = v_1' + v_2'$$

4. Calculate speed of puck #2 after collision in same way that speed of puck #1 was calculated earlier.

5. Show that data verifies the conservation of momentum by substituting values into speed equation developed in Step 3.

DATA TABLE
Before Collision
Distance between marks 1 and 2 _____
Time for puck #1 to travel between marks 1 and 2 _____
Average speed of puck #1 before collision v_1 _____
Speed of puck #2 before collision $v_2 = 0$ _____
After Collision
Distance between marks 3 and 4 _____
Time for puck #2 to travel between 3 and 4 _____
Average speed of puck #2 after collision v_2' _____
Speed of puck #1 after collision v_1' _____

ANGULAR MOMENTUM

MATERIALS

One rotating platform (or stool) (Cenco #74790-000 or equivalent). Two 2 kg masses.

PROCEDURE

Stand on a rotating platform (or sit on a rotating stool) with a 2 kg mass or more in each hand. Have a laboratory partner or instructor start the platform rotating. Change the position of your hands from a tucked-in position at your sides to a fully extended position, and observe the change in angular velocity.

How do your observations compare with the discussion of angular momentum presented in this module?

ELASTIC AND INELASTIC COLLISIONS

MATERIALS

One steel base, 2" x 2" x 1/2" or larger.
Equal-sized spheres (marble-size) of different materials (glass, steel, copper, lead, putty).

PROCEDURE

Obtain a relatively large, flat piece of steel. From identical heights, drop equal-sized spheres made of glass, steel, copper, lead, and putty onto the steel base. Measure the rebounding height of each sphere. On an arbitrary scale of 0 to 10, rate each collision, with 0 being an almost perfect inelastic collision (kinetic energy not being conserved) and 10 being an almost perfect elastic collision (kinetic energy completely conserved).

SELF CHECK

1. Define the following terms; state both the mk/s and the cg/s units for each; and give the equation for each.

   Linear momentum.
   Moment of inertia.
   Angular velocity.
   Angular momentum.
   Impulse.
   Angular impulse.

2. What is the momentum of a 250-lb object moving with a velocity of 5 ft/s? (Hint: Be sure to convert 250-pound weight to mass in slugs.)

3. If a rifle were lighter than the bullet it fired, which would be safer — the rifleperson or the target? Explain your answer in terms of the law of conservation of momentum.

4. A proton (mass = 1.67×10^{-27} kg) moves with a velocity of 5×10^{6} m/s. Upon colliding with a stationary particle of unknown mass, the proton rebounds along its original path with a velocity of 3×10^{6} m/s. The collision sends the unknown particle forward with a velocity of 1.5×10^{6} m/s. What is the mass of the unknown particle?

5. A projectile with a mass of 0.5 kg and a velocity of 700 m/s collides with a 20-kg mass initially at rest. The projectile imbeds itself within the mass. What is the velocity of the 20-kg mass and projectile after the collision? (Remember this: Momentum before equals momentum after!)

6. The radius of a solid shaft is 10 cm, and its mass is 5 kg. It is rotating with an angular velocity of 1800 r/min (rpm) about an axis of rotation through its center, as is shown in Figure 6b.

   a. What is the angular momentum of the shaft?
   b. What angular impulse is needed to increase the angular velocity to 2400 r/min?
   c. What constant torque is needed to increase the angular velocity from 1800 r/min to 2400 r/min in 30 seconds?

7. An Olympic high diver changes his speed of rotation while in flight by tucking himself into a small ball or by straightening himself out. Explain why this is possible in light of the conservation of angular momentum.

8. Use Equation 8 to calculate the force produced on one blade of a reaction turbine when 5 kg per second of water strikes the blade with a velocity of 7 m/s.

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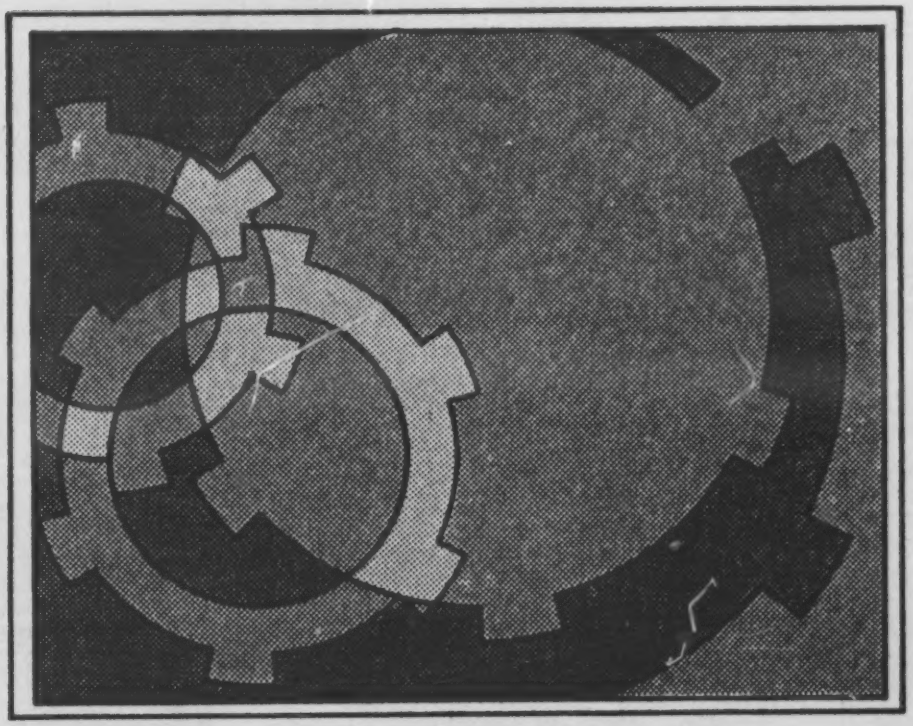
UNIFIED TECHNICAL CONCEPTS

MODULE 4 M 1

ANGULAR MOMENTUM OF A FLYWHEEL



MECHANICAL APPLICATION OF MOMENTUM

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INTRODUCTION

In any mechanical system involving rotational motion, angular momentum must be considered. One common, direct application of the concept of angular momentum is the flywheel. Flywheels are employed in most heat engines (internal combustion, steam) and in other types of mechanical systems where smooth, constant, rotary motion is desirable, such as tape players and stereo turntables.

In this module, the student will examine the concept of angular momentum. Angular momentum of several rotating systems will be calculated. In the laboratory, the angular momentum of a flywheel will be measured by two techniques and the results compared.

PREREQUISITES

Student should have completed Concept Module 4-0, "Momentum," and Application Modules 1M2, "Addition of Torques," and 3M2, "Velocity at a Constant Acceleration."

OBJECTIVES

To complete this module, the student should be able to:

- Define:
  - Moment of inertia.
  - Angular momentum.
- Given mass, diameter, angular velocity and formulas, calculate the angular momentum and moment of inertia of a uniform disk and of a wheel with all its mass near its rim.
- Calculate angular momentum of a planetary body in circular orbit, given its mass, velocity and radius of its orbit.
- Determine angular momentum of a wheel by two methods and compare the results.

DISCUSSION

Before examining the concept of angular momentum, it is necessary to have an understanding of equations and quantities involved in describing circular motion. Table I lists some of the concepts already studied and introduces some new concepts. The first five entries should be familiar.

TABLE I: ANALOG BETWEEN TRANSLATIONAL AND ROTATIONAL QUANTITIES

Concept	Translation	Rotation	Comments
1. Displacement	s	θ	$s = r\theta$
2. Velocity	$v = \frac{\Delta s}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$	$v = r\omega$
3. Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$	$a = r\alpha$
4. Equations of motion	$v_2 = v_1 + at$ $s = v_1 t + 1/2\ at^2$ $v_2^2 = v_1^2 + 2as$	$\omega_2 = \omega_1 + \alpha t$ $\theta = \omega_1 t + 1/2\alpha t^2$ $\omega_2^2 = \omega_1^2 + 2\alpha\theta$	acceleration constant
5. Force, torque	$F = \frac{\Delta P}{\Delta t}$	$\tau = \frac{\Delta L}{\Delta t}$	$\tau = Fr$
6. Mass, moment of inertia	$m = \frac{F}{a}$	$I = \frac{\tau}{\alpha}$	
7. Momentum	$P = mv$	$L = I\omega$	$L = Pr$

In entry six, mass is defined as the force applied to an object divided by the acceleration caused by that force. The analogous concept in circular motion is called the moment of inertia. Moment of inertia is defined by Equation 1.

$$\text{Moment of inertia} = \frac{\text{torque}}{\text{angular acceleration}} \qquad (1)$$

In symbols: $I = \tau/\alpha$.

In the next entry, linear moment is defined as the product of mass and velocity. The corresponding concept of angular momentum is defined by Equation 2.

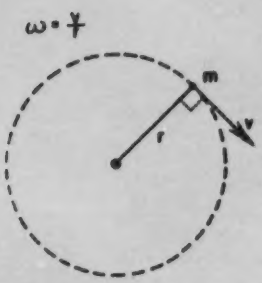
Angular moment = moment of inertia x angular velocity (2)

As $v = \omega r$.

In symbols: $L = I\omega$.

Figure 1 shows a particle of mass m traveling with velocity v in an orbit of radius r. The linear momentum is mv. Angular momentum is the product of the linear momentum and the radius of the orbit.

$\omega = \frac{v}{r}$.



$$L = mvr$$
$$= m(r\omega)r$$
$$L = mr^2\omega$$

From Equation 2:

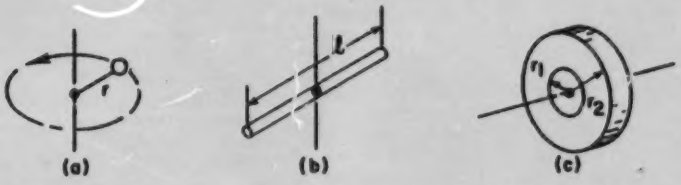
$$I = \frac{L}{\omega}$$
$$= \frac{mr^2\omega}{\omega}$$
$$I = mr^2$$

As $L = mr^2\omega$.

Fig. 1

Thus, the moment of inertia of a particle in a circular orbit is mr^2. Using this formula, it is possible to determine the moment of inertia of any uniform object. Figure 2 gives the moments of inertia of several common shapes.

Fig. 2



(a) Particle in circular orbit

$I = mr^2$

(b) Slender rod, axis through center

$I = \frac{1}{12} m\ell^2$

(c) Annular cylinder

$I = \frac{1}{2} m (r_1^2 + r_2^2)$

(d) (e) (f)

Fig. 2 (cont)

Solid cylinder
$I = \frac{1}{2} mr^2$

Thin-walled hollow cylinder (wheel)
$I = mr^2$

Solid sphere
$I = \frac{2}{5} mr^2$

Example A shows the use of Figure 2 and Equation 2 in solving a problem.

EXAMPLE A: ANGULAR MOMENTUM OF A DISK

Given: A uniform disk of radius 10 cm and mass 2 kg rotating about an axis through its center with an angular velocity of 0.5 rad/s.

Find: Its angular momentum.

Solution:

$L = Iw$

$= (\frac{1}{2}mr^2)\, w$ since $I = \frac{1}{2}mr^2$

$= (0.5)(2\ kg)(0.1\ m)^2(0.5/s)$

$L = 0.005\ \frac{kg \times m^2}{s}$

Angular momentum = $0.005\ \frac{kg \times m^2}{s}$

Example B shows use of the equations in Table I in solving a more complex problem.

EXAMPLE B: FINAL ANGULAR MOMENTUM OF A DISK AND MASS

Given: The solid disk shown with a small pin on its circumference that holds a string in place at point A. The string is attached to a 0.1-kg mass. When the disk is released, it will begin to turn as the weight falls. The string will slip off the pin when the pin reaches point B. When this was done in the laboratory, 0.67 seconds elapsed between the release of the wheel and the pin reaching point B.

Find: Final angular momentum of the disk.

Solution A:

$\omega_1 = 0$ rad/s

$t = 0.67$ s

$g = 9.8$ m/s^2

As the weight falls, it falls through a distance ℓ.

$\ell = v_1 t + 1/2\ at^2$

but $v_1 = 0$, therefore $v_1 t = 0$

$\ell = 1/2\ gt^2$

$a = \frac{2\ell}{t^2}$

$= \frac{2\ (0.2\ m)}{(0.67\ s)^2}$

$a = 0.891$ m/s^2

Because pin and mass are tied together, their acceleration is the same.

$\alpha = \frac{a}{r}$

$= \frac{0.891\ m/s^2}{0.15\ m}$

Angular acceleration of the disk

$\alpha = 5.94$ rad/s^2

$\omega_2 = \omega_1 + \alpha t$

but $\omega_1 = 0$

$\omega_2 = \alpha t$

$= (5.94\ rad/s^2)(0.67\ s)$

Final angular velocity of disk

$\omega_2 = 3.98$ rad/s

Remember that "radian" is not a unit and can be dropped.

$I = 1/2\ mr^2$

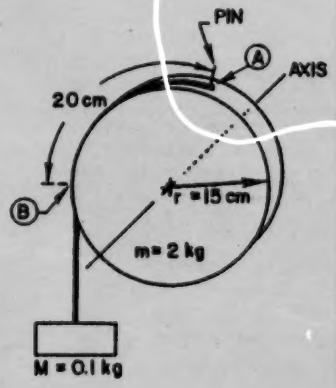
$= (0.5)(2\ kg)(0.15\ m^2)$

$I = 0.0225\ kg \times m^2$

$L = I\omega$

$= (0.0225\ kg \times m^2)(3.98\ rad/s)$

$L = 0.0895\ \frac{kg \times m^2}{s}$



Solution B:

The problem can be solved by an alternate method if $T = \frac{\Delta L}{\Delta t}$ from Table I is used. This gives:

$L = \tau t$

$\tau = Fr$, where F is force applied at point A

$F = MG - Ma$

$= M(G - a)$

$= (0.1\ kg)(9.8\ \frac{m}{s^2} - 0.891\ \frac{m}{s^2})$

$F = 0.891\ \frac{kg \times m}{s^2}$

$\tau = Fr$

$= (0.891\ \frac{kg \times m}{s^2})(0.15\ m)$

$\tau = 0.1337\ \frac{kg \times m^2}{s^2}$

$L = \tau t = (0.1337\ \frac{kg \times m^2}{s^2})(0.67\ s)$

$L = 0.0896\ \frac{kg \times m^2}{s}$

Thus, the problem can be solved by either method.

MG is gravitational force applied. Ma is the portion of that force used to accelerate that weight.

MATERIALS

Bicycle wheel with tire.
100-g weight.
1-m length of string.
Meter stick.
Laboratory timer or stopwatch.
Short, headless pin.
Spring scale.

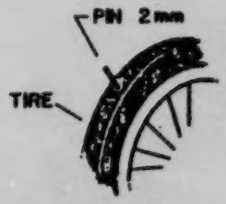


Fig. 3

PROCEDURE

1. Using scale, determine mass of bicycle wheel. Assume that all mass is located at rim. Record mass in data table.
2. Measure and record radius of wheel.
3. Insert pin into tire as shown in Figure 3, perpendicular to tire surface and with only about 2 mm exposed.
4. Tie a loop in one end of string. Measure and mark a distance of 50 cm from loop. Tie 100-g weight to other end of string.
5. Arrange equipment as shown in Figure 4.
6. Simultaneously release wheel and start timer. Stop timer when string slips from pin. You may need to do this several times to get a good average value of time span.
7. Record all data; complete calculations portion of data table. (Refer to Example B.)

DATA TABLE

DATA

Mass of bicycle wheel: m = _____ kg

Mass of weight: m = _____ kg

Radius of wheel: r = _____ m

Distance for uniform acceleration: s = _____ m

Time for uniform acceleration: t = _____ s

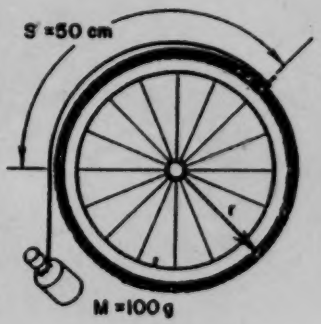


Fig. 4

CALCULATIONS

Moment of inertia: $I = mr^2 =$ _____ kg x (_____ m)^2

$I =$ _____ kg x m^2.

Acceleration of string: $a = \frac{2s}{t^2}$

$a = 2 \times$ _____ m $\div$ (_____ s)^2

$a =$ _____ m/s^2.

Angular acceleration: $\alpha = \frac{a}{r}$

$\alpha =$ _____ m/s^2 $\div$ _____ m

$\alpha =$ _____ rad/s^2.

Final angular velocity: $\omega = \alpha t$

$\omega =$ _____ rad/s^2 x _____ s

$\omega =$ _____ rad/s.

Angular momentum (first method): $L_1 = I\omega$

$L_1 =$ _____ kg x m^2 x _____/s

$L_1 =$ _____ kg x m^2/s.

Tangential force on wheel: $F = M(g - a)$

$F =$ _____ kg (9.8 m/s^2 - _____ m/s^2)

$F =$ _____ kg x m/s^2.

Torque: $\tau = Fr$

$\tau =$ _____ kg x m/s^2 x _____ m

$\tau =$ _____ kg x m^2/s^2.

Angular impulse: $L_2 = \tau t$

$L_2 =$ _____ kg x m^2/s^2 x _____ s

$L_2 =$ _____ kg x m^2/s.

Recall from Module 5-0:

Angular impulse = angular momentum

$$\% \text{ variation} = \frac{\text{angular impulse} - \text{angular momentum}}{\text{average}} \times 100.$$

SELF CHECK

1. Define:
   Moment of inertia.
   Angular momentum.
2. A uniform disk has a mass of 3.6 kg and a diameter of 0.6 m. What is the moment of inertia? ($I = 1/2\ mr^2$.)
3. A wheel of radius 12 cm has a mass of 0.68 kg. If all mass is considered to be at the rim, what is its moment of inertia? ($I = mr^2$.)
4. A 100-g weight is swung at constant speed in a circle at the end of the 1-m string. It takes 1.2 seconds for one complete revolution. What is its angular momentum?
5. A dish 32 cm in diamter has a mass of 4.8 Kg. It is spinning at 1500r/min. Calculate its' angular momentum using $L = Iw$ and $L = \tau t$ and compare the results.

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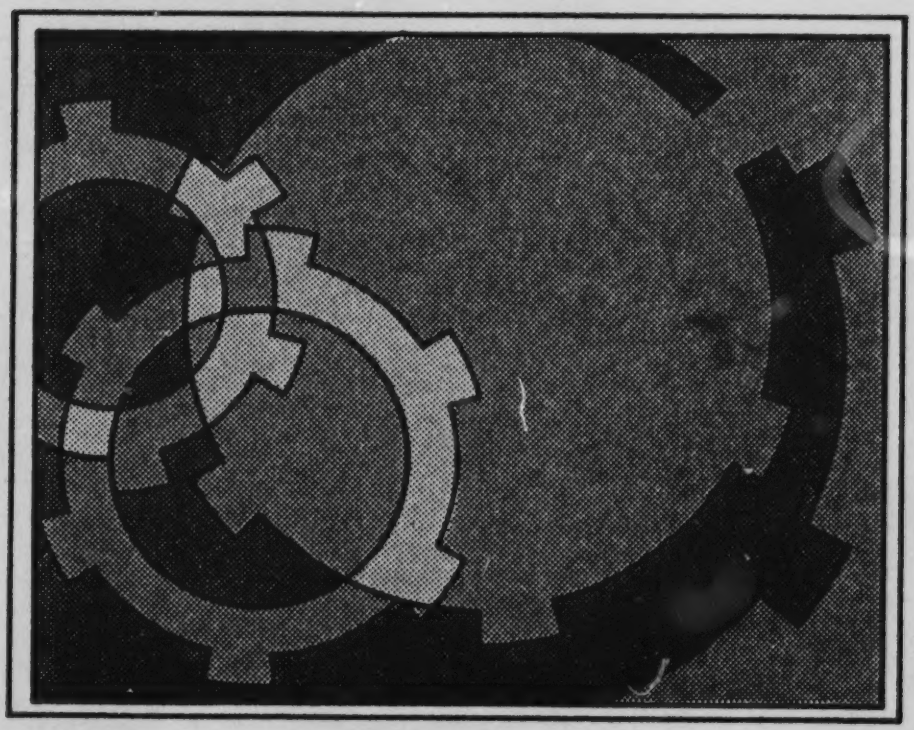
UNIFIED TECHNICAL CONCEPTS

MODULE 4 M 2

IMPULSE AND MOMENTUM



MECHANICAL APPLICATION OF MOMENTUM

Technical Education Research Center - Southwest

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INTRODUCTION

Impulse is the product of force and the duration of application of the force. An impulse applied to a mass causes a change in momentum of the mass. Excessive applied forces may deform the object permanently. Application of smaller forces over a longer duration can achieve the desired change in momentum, but with reduced mechanical stress. This module discusses impulse and change in momentum as it applied to mechanical components and systems of modern technology. In the laboratory, the student will measure impulse and change in momentum.

PREREQUISITES

Student should have completed Concept Module 4-0, "Momentum."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define impulse and momentum.
- Derive the relationship between impulse and momentum from Newton's second law.
- Given three of the following quantities, calculate the fourth:

  Mass of object.
  Change in velocity.
  Applied force (constant).
  Duration of application of force.

- Estimate the impulse from a graph of force-versus-time by approximating the area under the force-versus-time curve.
- Explain the importance of considering impulse in the design of machine parts.

- Given the appropriate equipment, measure the impulse and momentum change when a force accelerates a mass.

DISCUSSION

Newton's second law (Equation 1) states the relationship of a constant force, mass, and acceleration as follows:

$$F = ma \qquad (1)$$

where

F = force.
m = mass.
a = acceleration.

An applied force causes an acceleration of a mass. Acceleration is the change in velocity divided by the time interval over which that change occurs. Thus, Newton's second law may be written in the form of Equation 2.

$$F = m\frac{\Delta v}{\Delta t} \qquad \text{(substituting } a = \frac{\Delta v}{\Delta t}\text{)}$$

or

$$F\Delta t = m\Delta v \quad \text{(multiplying each side by } \Delta t\text{)} \qquad (2)$$

where

Δv = change in velocity.
Δt = time interval over which Δv occurs.

The quantity of the left side of Equation 2 is called the "impulse." It is the product of the applied force and the duration of action of that force. "Momentum" is the product of the mass of an object and its velocity. The quantity on the right side of Equation 2 is mass time change in velocity and represents change in momentum. Thus, Equation 2 may be stated as —

Impulse = Change in momentum.

Notice that both impulse and momentum are vector quantities. Examples A and B show the use of Equation 2 in solving problems:

EXAMPLE A: CALCULATION OF CHANGE IN VELOCITY	
Given:	A force of 10 N is applied to a 25-kg mass for a time of 2 s.

Find: (a) Impulse
(b) Change in velocity.

Solution: (a) Impulse is defined as $F\Delta t$.

$$F\Delta t = m\Delta v$$
$$= (10\ N)(2\ s)$$
$$F\Delta t = 20\ N \cdot s, \text{ to the right}$$

(b) $F\Delta t = m\Delta v$

$$\Delta v = \frac{F\Delta t}{m}$$
$$= \frac{20\ N \cdot s}{25\ kg}$$
$$= 0.8\ \frac{\frac{kg \cdot m}{s^2} \cdot s}{kg}$$
$$\Delta v = 0.8\ m/s, \text{ to the right}$$

EXAMPLE B: CALCULATION OF DURATION OF FORCE

Given: A constant force of 5 N is applied to a 25 kg mass and changes its velocity by 10 m/s.

Find: The duration of the applied force.

Solution: $F\Delta t = m\Delta v$

$$\Delta t = \frac{m\Delta v}{F}$$
$$= \frac{(25\ kg)(10\ m/s)}{5\ kg \cdot m/s}$$
$$\Delta t = 50\ s$$

Figure 1 represents a constant force F_1 applied to an object and the velocity of that object as functions of time. The impulse of the force is the area $F_1\Delta t_1$. This impulse causes a momentum change of $m\Delta v$, where m is the mass of the object and Δv is the change in its velocity. The acceleration is the slope of the velocity of $\frac{\Delta v}{\Delta t_1}$.

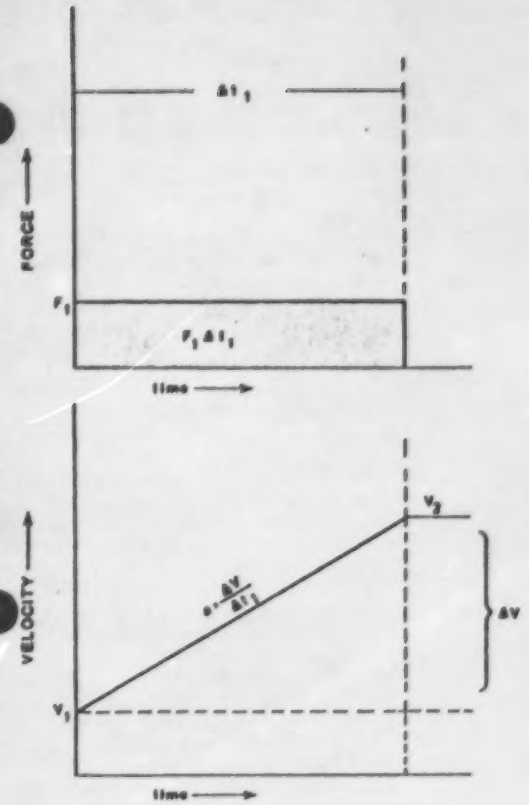


FIG. 1

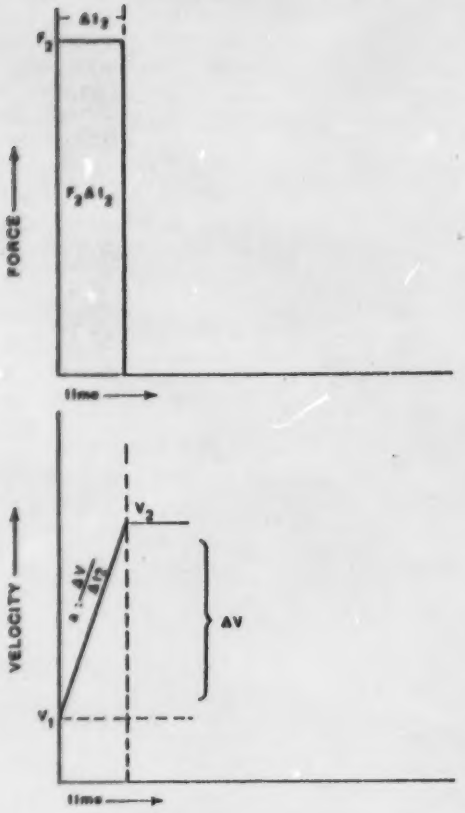


FIG. 2

Figure 2 shows a larger force F_2 applied for a shorter period Δt_2 to produce the same total impulse and the same net change in momentum ($m\Delta v$). Equal areas (shaded) under the force-versus-time curves represent the same impulse and cause the same change in momentum. If an applied force F is increased a hundred times (to 100F) and the time of application Δt is decreased to $\Delta t/100$, the impulse $F\Delta t$ equals $100F \frac{\Delta t}{100}$, so that both the impulse and the change in momentum remain the same.

The stress on the object being accelerated is proportional to the applied force. If the applied force is large enough, deformation

may occur, although the change in velocity remains the same. A machine brought up to speed rapidly may be over-stressed. Reduction of the force and increase in the duration of application can achieve the desired change in momentum, but with reduced stress.

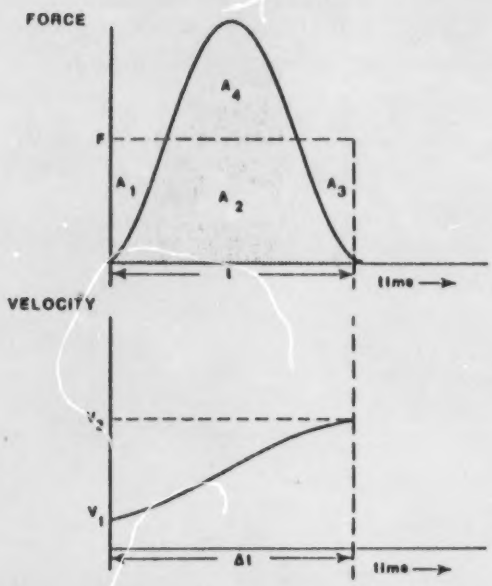


FIG. 3

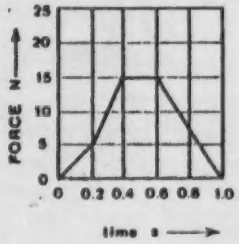
A machine whose motion is described in Figure 1 has an initial velocity v_1. At the instant the force F_1 is applied, the acceleration changes, almost abrubtly, from zero to $a = F_1/m$. This effect is experienced as "jerk," which can be eliminated by gradually increasing the force — and, thus, the acceleration — from zero to the desired level.

The accelerating force applied to an elevator, for example, changes smoothly from zero to maximum and back to zero to reach the desired velocity, without the abrupt change in acceleration. This situation is shown in Figure 3. The impulse is the same as in Figures 1 and 2; but in the curve described in Figure 3, there is no abrupt jerk in the motion. The impulse depicted in Figure 3 is the sum of the areas $(A_2 + A_4)$. The impulse associated with a constant force F_1 (dotted line in Figure 3) is the sum of the areas $(A_1 + A_2 + A_3)$. These impulses are equal if $(A_1 + A_3 = A_4)$.

Example C demonstrates how to determine the change in the momentum of an object from an impulse graph.

EXAMPLE C: IMPULSE GRAPH

Given: A 10-kg object initially at rest is subjected to the force described in the graph shown.



Find: Final velocity of the mass.

Solution: The area of one square of the graph is 5N x 0.20 s = 1 N·s. The area under the impulse curve is 8.5 squares.

$$\text{Impulse} = 8.5 \text{ squares} \times 1 \text{ N·s/square}$$

$$F\Delta t = 8.5 \text{ N·s}$$

$$F\Delta t = m\Delta V$$

$$\Delta v = \frac{F\Delta t}{m}$$

$$= \frac{8.5 \text{ kg·m/s}}{10 \text{ kg}}$$

$$\Delta v = 0.85 \text{ m/s}$$

In an automotive engine, a cam working against a valve spring smoothly changes the acceleration of the valve. If the acceleration is too large or abrupt, the valve may "float" and not remain in contact with the cam, causing damage to the engine. Use of the proper cam shape and the maintaining of engine speed below the design limit decreases the applied force (and acceleration) and increases the duration of acceleration.

Newton's third law states that for every action, there is an equal and opposite reaction; therefore, if object A imparts an impulse to B, then B imparts an equal and opposite impulse to A. This law indicates that deformation of both objects may occur if the stress produced exceeds their elastic limits. Care must be taken to avoid this in all mechanical systems. Aircraft wings are subject to variable impulse loadings during flight, due to turbulent air or landings. Little can be done about the magnitude of the impulse during these situations; however, the force element of impulse can be reduced by allowing the wings to flex, thereby increasing the duration of the impulse. This controlled deformation serves as a shock absorber. Aircraft landing gears are designed to include shock-absorbing systems that for a given impulse increases the duration Δt of the applied force and, consequently, reduces the magnitude of the applied force. The elasticity of the shroud lines of a parachute serve this shock absorbing function and make the sudden opening of the chute and reduction of momentum more tolerable.

In crash situations, final momentum will be reduced to zero.

The impulse transmitted to the occupants, therefore, is equal to their initial momentum. The time must be lengthened to reduce the force applied to the occupants. This lengthening of time can be done, to some extent, by designing the vehicle to deform in a controlled manner. (Incidentally, inflatable air bags between the driver and the automobile frame are designed to do just that.)

Fluids in motion also possess momentum. The fluid in a pipeline may have a very large momentum because of its mass. If a valve is closed suddenly, the impusle applied to the valve by the change in momentum of the fluid may rupture the pipe at the valve head. For this reason, the flow in a large pipeline is shut off slowly. When water in the home is turned off quickly, the pipe may make a hammering noise because of the large forces resulting when the momentum of the flowing water is changed in a short time.

In rotational systems, the corresponding quantities are angular impulse and angular momentum. The angular impulse is the product of the torque and the duration of the application of that torque and is equal to the change in angular momentum. The same general principles apply to rotational systems for reducing wear and the probability of mechanical failure. The clutch or automatic transmission of an automobile slips upon the initial application of the engine torque to the drive train. This slippage reduces the torque in the drive train by allowing the momentum changes to occur over a longer period of time.

SUMMARY

When a force acts to accelerate an object, the velocity of the object and its momentum change. The impulse of the force is the product of the magnitude of the force and the duration of its application and is equal to the change in momentum of the object. Mechanical parts experience strain and wear proportional to the forces applied to them. When the momentum of a part is changed, minimum wear occurs if a minimum force is used to make that change. A common consideration in all areas of mechanical design is the reduction of strain through necessary momentum changes by applying smaller forces for longer times.

IMPULSE AND CHANGE IN MOMENTUM

MATERIALS

Airtrack.
Cart.
Photocell timer.
String.
Pulley.
Weight.
Balance.
Meterstick.

PROCEDURE

1. Measure and record length (ℓ) and mass (m_1) of cart and mass (m_2) of weight.

2. Set up equipment as illustrated in Figure 4. Photocell timer should be positioned in such a manner that weight hits floor before cart reaches timer. Be sure air track is level.

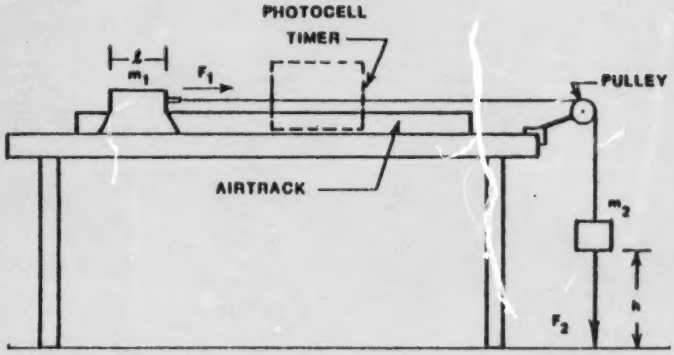


FIG. 4

3. Hold cart in place. Measure and record height (h) of weight.

4. Release cart. Record time (t) for cart to pass through photocell gate.

5. Complete calculations in Data Table to determine impulse and final momentum of cart.

6. Draw a graph of force versus time for this experiment, and show that the area under the curve represents both the impulse and the change in momentum.

7. If m_2 were three times as great, how would the results be affected?

DATA TABLE

Data:

Mass of cart: m_1 = _____

Mass of weight: m_2 = _____

Length of cart: ℓ = _____

Height of weight: h = _____

Time of cart through gate: t = _____

Calculation of Impulse:

Force causing motion of system: $F_2 = m_2 g$ = _____

Acceleration of system: $a = \frac{F}{m} = \frac{m_2 g}{(m_1 + m_2)}$ = _____

Force on cart: $F_1 = (m_1)\left(\frac{m_2 g}{m_1 + m_2}\right) = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g$

F_1 = _____

Time for acceleration: $\Delta t = \frac{2h}{a}$ = _____

Impulse: $F_1 \Delta t$ = _____

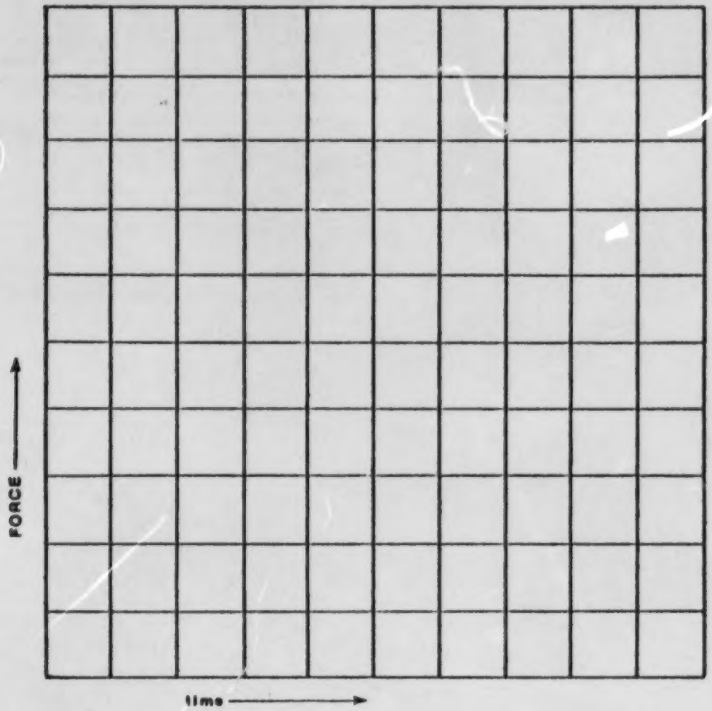
Calculation of Momentum Change:

Velocity of Cart: $v = \frac{\ell}{t}$ = _____

Final momentum of cart: $p = mv$ = _____

Momentum change = Δmv = _____

Impulse _____ Δ Momentum _____



SELF CHECK

1. Define:

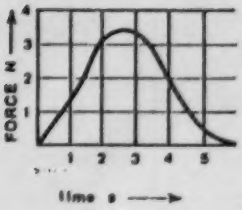
   a. Impulse.
   b. Momentum.

2. Derive the relationship of impulse to momentum change from the following:

   Force = Mass x Acceleration

3. A force of 16N acts on a 2-kg mass for 3.5s. Determine the change in the velocity of the mass by —

   a. Finding its momentum change.
   b. Finding the acceleration of the mass.



4. The force described by the adjoining graph was applied to an object initially at rest. The area under the curve is measured to be roughly 10.5 Ns. Its final velocity was 16 m/s. Find its mass.

5. How long must a 50 N force be applied to a 25-kg mass to change its momentum by 1000 kg·m/s?

6. Describe the importance of considering impulse when designing parts for high-speed mechanical systems.

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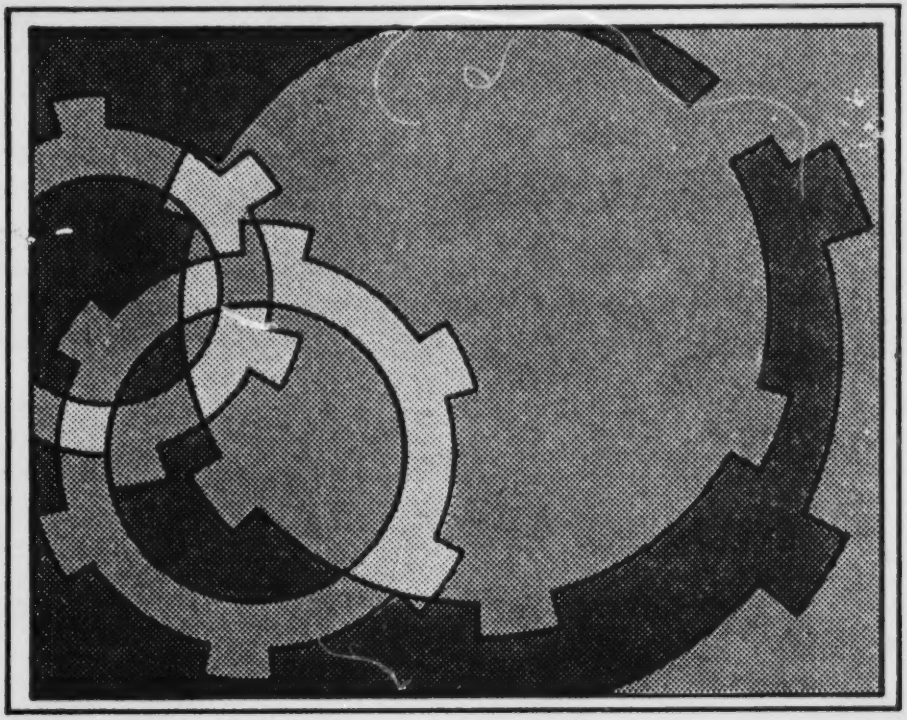
UNIFIED TECHNICAL CONCEPTS

MODULE 4 M 3

CONSERVATION OF MOMENTUM IN COLLISION



MECHANICAL APPLICATION OF RATE

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

Collisions are one of the most common physical phenomena that occur naturally or as a result of modern technology. The effects of collision can be determined from the law of conservation of momentum. This law is valid in isolated systems in which no external forces act. The principle of "the conservation of momentum" applies to such an inconsequential event as the collision of two billiard balls; to basic physical phenomena such as collisions between atoms; and to more catastrophic collisions such as those between automobiles. The study of collisions between subatomic particles is a powerful tool in understanding the structure of matter.

In this module, momentum conservation is applied to elastic and inelastic collisions. In the laboratory, the student will analyze both elastic and inelastic collisions in one dimension to verify conservation of momentum.

PREREQUISITES

Student should have completed Concept Module 4-0, "Momentum."

OBJECTIVES

Upon completion of this module, the student should be able to:

- State the law of conservation of momentum mathematically and verbally.
- Define and give two examples of these collisions:

  Elastic collision.
  Inelastic collision.

- Apply the principle of conservation of momentum to solve practical examples of one-dimensional collisions, using the following procedures:

  Draw and analyze a vector diagram of the collision.
  Solve the conservation of momentum equation.

- Given the appropriate equipment, analyze elastic and inelastic collisions in one dimension.

DISCUSSION

In a collision between the cue ball and the eight ball in billiards, some of the cue ball's momentum is transferred to the eight ball. Generally, both balls move off in different directions and with less speed than the cue ball had possessed initially. If the collision is head-on and if no energy is lost during the collision, all the cue ball's momentum is transferred to the eight ball, and the cue ball stops.

When a coasting caboose couples with an empty, stationary freight car on a railroad track, the caboose and the freight car move off together at a reduced speed. Momentum is transferred between two automobiles when they collide. A proton, scattering off a heavy nucleus at rest, imparts some of its momentum to the nucleus and moves off in a different direction at slower speed. In all of these examples, conservation of momentum applies in the absence of external forces.

The collisions discussed above may be classified as either "elastic" or "inelastic." An elastic collision is one in which kinetic energy is conserved; whereas, in inelastic collisions, kinetic energy is not conserved. Some of the energy of motion is transferred into another form of energy. When the colliding bodies stick together after the collision, the collision is completely inelastic.

The collision between the caboose and freight car is completely inelastic, since the cars couple; kinetic energy is used to couple the cars. The collision between two automobiles is inelastic because kinetic energy is used to deform them and to produce heat. Such a collision is completely inelastic if the automobiles happen to stick together after impact. The collision between billiard balls can be considered elastic for practical purposes, although the balls are slightly deformed upon collision. The only truly elastic collisions are those between subatomic particles, such as the one involving the proton and the nucleus. Even these may be inelastic, however, when an atom is "excited" by an incident particle and absorbs some of its kinetic energy upon impact.

All collisions are governed by Newton's Laws. When two bodies collide, the forces exerted by the bodies on each other are equal and opposite. For example, the force exerted by the cue ball on the eight ball is of equal magnitude and in the opposite direction to that exerted by the eight ball on the cue ball. The mathematical statement of Newton's third law is simply $\vec{F} = -\vec{F}$. In addition, the force on either body is given by Newton's second law:

$$\vec{F} = m\vec{a}$$

or, Force = mass x acceleration.

If the time elapsed during collision is Δt, then the following relationship can be derived:

$$\vec{F}_1 = -\vec{F}_2 \qquad \text{(Newton's third law)}$$

$$\vec{F}_1 \Delta t = -\vec{F}_2 \Delta t \qquad \text{(multiplying both sides by } \Delta t\text{)}$$

$$m_1 \vec{a}_1 \Delta t = -m_2 \vec{a}_2 \Delta t \qquad \text{(Newton's second law, } \vec{F} = m\vec{a}\text{)}$$

$$m_1 \frac{\Delta v_1}{\Delta t} \Delta t = -m_2 \frac{\Delta v_2}{\Delta t} \Delta t \qquad \left(\text{since } \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time elapsed}}\right)$$

$$m_1 \Delta \vec{v}_1 = -m_2 \Delta \vec{v}_2.$$

The change in velocity $\Delta \vec{v}$ is defined as the velocity after collision, minus the velocity before collision. Velocities before collision will be unprimed; those after collision will be primed. Thus —

$$\Delta \vec{v} = \vec{v}_1' - \vec{v}_1$$

Then —

$$m_1 (\vec{v}_1' - \vec{v}_1) = -m_2 (\vec{v}_2' - \vec{v}_2)$$

$$m_1 \vec{v}_1' - m_1 \vec{v}_1 = -m_2 \vec{v}_2' + m_2 \vec{v}_2.$$

The quantity $(m_1 \vec{v}_1 + m_2 \vec{v}_2')$ now is added to both sides of the equation to form Equation I.

$$m_1 \vec{v}_1' - m_1 \vec{v}_1 + m_1 \vec{v}_1 + m_2 \vec{v}_2' = -m_2 \vec{v}_2' + m_2 \vec{v}_2 + m_1 \vec{v}_1 + m_2 \vec{v}_2'$$

$$m_1 \vec{v}_1' + m_2 \vec{v}_2' = m_2 \vec{v}_2 + m_1 \vec{v}_1 \qquad \text{(I)}$$

The total momentum $\vec{P}$ of the system of two bodies before collision is indicated thusly:

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 \qquad (\text{since } \vec{p} = m\vec{v})$$

Likewise, after the collision, the total momentum is —

$$\vec{P}' = \vec{p}_1' + \vec{p}_2'$$

$$\vec{P}' = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

Therefore, Equation I is equivalent to —

$$\vec{P} = \vec{P}'$$

or Momentum before collision = Momentum after collision.

For completely inelastic collisions in which the colliding bodies stick together, the final velocities $\vec{v}_1'$ and $\vec{v}_2'$ are equal; and —

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v} + m_2\vec{v} \qquad (\text{where } \vec{v}_1' = \vec{v}_2' = \vec{v})$$
$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}.$$

If one body is initially at rest, such as when the cue hits the eight ball —

$$m_1\vec{v}_1 + m_2(0) = m_1\vec{v}_1' + m_2\vec{v}_2' \qquad (\text{since } \vec{v}_2 = 0)$$
$$m_1\vec{v}_1 = m_1\vec{v}_1' + m_2\vec{v}_2'.$$

If the collision is between bodies of equal mass, such as billiard balls, Equation I simplifies as follows:

$$m\vec{v}_1 + m\vec{v}_2 = m\vec{v}_1' + m\vec{v}_2' \qquad (\text{where } m_1 = m_2 = m)$$
$$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$$

A head-on collision is one-dimensional; therefore, the vector sign can be dropped:

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

It should be noted, however, that the velocities v_1, v_2, v_1', and v_2' can be positive or negative.

Examples A, B, C, D, and E show the use of Equation I in solving both elastic and inelastic collision problems in one dimension.

EXAMPLE A: ONE-DIMENSIONAL ELASTIC COLLISION OF EQUAL MASSES

Given: Identical marbles collide head-on. Marble A is traveling at 0.5 m/s initially and stops on impact. Marble B initially is at rest. Assume a perfectly elastic collision.

Find: The velocity of Marble B after collision. Also show that the kinetic energy of the marbles is the same before and after the collision.

Solution: Because the marbles are identical, their masses are equal:

$v_1 = 0.5$ m/s $\qquad v_1' = 0$

$v_2 = 0 \qquad\qquad v_2' = ?$

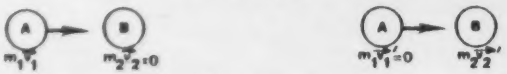
Momentum before collision = Momentum after collision.

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$v_1 + v_2 = v_1' + v_2' \text{ (since } m_1 = m_2\text{)}$$

$$v_1 = v_2' \qquad \text{(since } v_2 = 0 \text{ and } v_1' = 0\text{)}$$

$$v_2' = 0.5 \text{ m/s}$$



Also, since $m_A = m_B$ and $v_2' = 0.5$ m/s $= v_1$, it is clear that $1/2\ m_A(0.5)^2 = 1/2\ m_B(0.5)^2$; so kinetic energy is conserved.

EXAMPLE B: ONE-DIMENSIONAL ELASTIC COLLISION OF UNEQUAL MASSES

Given: A marble of mass 50 g moving at 0.4 m/s collides head-on with a smaller marble of mass 30 g at rest. The final velocity of the large marble is 0.10 m/s in the forward direction.

Find: The final velocity of the small marble. Show that kinetic energy is conserved.

Solution: The conditions given are:

$m_1 = 50$ g $\qquad v_1 = 0.4$ m/s $\qquad v_1' = 0.1$ m/s

$m_2 = 30$ g $\qquad v_2 = 0 \qquad\qquad v_2' = ?$

Conservation of momentum is applied:

Momentum before collision = Momentum after collision.

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$m_1v_1 = m_1v_1' + m_2v_2' \quad (\text{since } v_2 = 0)$$

$$(0.050\ \text{kg})(0.4\ \text{m/s}) = (0.050\ \text{kg})(0.1\ \text{m/s}) + (0.030\ \text{kg})\ v_2'$$

$$0.020\ \frac{\text{kg·m}}{\text{s}} = 0.005\ \text{kg-m/s} + (0.020\ \text{kg})\ v_2'$$

$$v_2' = \frac{(0.020 - 0.005)\ \text{kg-m/s}}{0.030\ \text{kg}}$$

$$v_2' = 0.5\ \text{m/s}$$

Conservation of kinetic energy is demonstrated thusly:

$$1/2\ (0.050)(0.4)^2 = 1/2\ (0.050)(0.1)^2 + 1/2\ (0.030)(0.5)^2$$

$$0.004\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2} = 0.00025\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2} + 0.00375\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}$$

$$\therefore\ 0.004\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2} = 0.004\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2} \quad \text{Q.E.D.}$$

EXAMPLE C: ONE-DIMENSIONAL ELASTIC COLLISION WITH BOTH MASSES ORIGINALLY IN MOTION

Given: A marble of mass 50 g moving at 0.4 m/s collides head-on with a marble of mass 20 g moving at -0.4 m/s. The final velocity of the large marble is -0.057 m/s; that is, the 50 g marble rebounds backwards.

Find: The final velocity of the small marble.

Solution: The conditions given are:

$m_1 = 50$ g $\quad v_1 = 0.4$ m/s $\quad v_1' = -0.057$

$m_2 = 20$ g $\quad v_2 = -0.4$ m/s $\quad v_2' = ?$

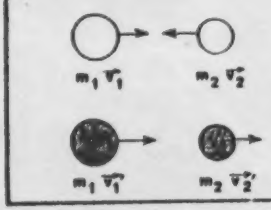
Conservation of momentum is applied.

Momentum before collision = Momentum after collision.

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$(0.050)(0.4) + (0.020)(-0.4) = (0.050)(-0.057) + (0.020)\ v_2'$$

$$0.020 - 0.008 = -0.00285 + (0.020)\ v_2'$$



$$0.01485 = 0.020\ v_2'$$

$$v_2' = \frac{0.01485}{0.02}$$

$$v_2' = 0.743$$

(The verification of the proper units is left as a student exercise.)

EXAMPLE 0: ONE-DIMENSIONAL INELASTIC COLLISION WITH ONE MASS STATIONARY

Given: A stationary refrigerator car of mass 2.0×10^5 kg is rammed by a loaded cattle car of mass 3.0×10^5 kg coasting at 1.0 m/s. The cars couple upon impact and coast together.

Find: Their final velocity.

Solution: Conservation of momentum for a completely inelastic collision applies.

Momentum before collision = Momentum after collision.

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v$$

The conditions given are:

$m_1 = 3.0 \times 10^5$ kg $\quad v_1 = 1.0$ m/s $\quad v = ?$

$m_2 = 2.0 \times 10^5$ kg $\quad v_2 = 0$

Then:

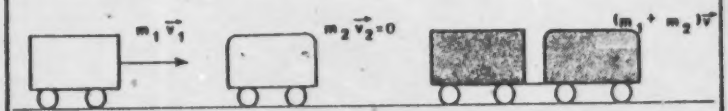
$$m_1 v_1 = (m_1 + m_2)v \qquad \text{(since } v_2 = 0\text{)}$$

$$(3.0 \times 10^5)(1.0) = (3.0 \times 10^5 + 2.0 \times 10^5)v$$

$$3.0 \times 10^5 = (5.0 \times 10^5)v$$

$$v = \frac{3.0 \times 10^5}{5.0 \times 10^5}$$

$$v = 0.6 \text{ m/s}$$



EXAMPLE E: ONE-DIMENSIONAL INELASTIC COLLISION WITH BOTH MASSES ORIGINALLY IN MOTION

Given: A northbound automobile weighing 910 kg traveling north at 24 m/s collides head-on with an automobile weighing 2050 kg traveling south at 28 m/s. The two cars stick together and coast at constant velocity for several meters.

Find: The velocity of the two cars just after collision.

Solution: If north is defined as the positive direction, then --

$m_1 = 910$ kg $\quad v_1 = 24$ m/s $\quad v = ?$

$m_2 = 2050$ kg $\quad v_2 = -28$ m/s

Momentum before collision = Momentum after collision.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

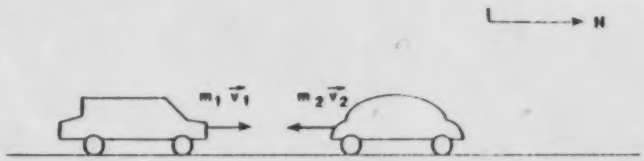
$$(910)(24) + (2050)(-28) = (910 + 2050)v$$

$$(-35560) = (2960)v$$

$$v = -\frac{35560}{2960}$$

$$= -12 \text{ m/s}$$

$v = 12$ m/s south



SUMMARY

The "conservation of momentum" is an important principle, which states that the total momentum of two colliding bodies is the same after the collision as before. Kinetic energy is conserved in "elastic" collisions but not in "inelastic" ones. All collisions are governed by Newton's laws. When two bodies collide, the forces exerted by the bodies on each other are equal and opposite. The force on either body is given by Newton's second law: Force equals mass times acceleration.

CONSERVATION OF LINEAR MOMENTUM

MATERIALS

Air track.
Two air carts with spring bumpers.
Spark timer with paper tape.
Putty.
Slotted masses.

PROCEDURE

Read all instructions carefully before beginning.

ELASTIC COLLISION:

1. Place one air cart near center of air track, and turn on blower. Level track so that cart does not move unless pushed.

2. Weigh both air carts, and record mass of each in Data Table I. (Be sure to use as target the cart whose mass is recorded as m_2.)

3. Position incident air cart (m_1) at one end of track and target cart (m_2) approximately at center of track, as in Figure 1. Place paper tape on cylinder of the spark timer, and start timer. Push incident cart slightly toward target. At instant of collision, rotate cylinder so that timer sparks on a different place on tape after collision. When either cart reaches end of track, turn off spark timer.

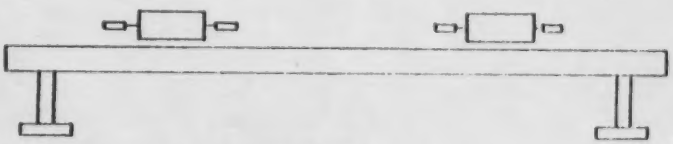


FIG. 1

4. Frequency of timer is the number of times it sparks each second; therefore, the period (time T between sparks) is

$$\text{period} = 1/\text{frequency}$$

$$T = 1/f.$$

Time t required for cart to travel a distance d then is

$$\text{time} = \begin{matrix}\text{number of time intervals in distance d} \\ \text{x time in each interval:}\end{matrix}$$

$$t = n \times T$$

$$t = n \times \frac{1}{f} \qquad (\text{since } T = \tfrac{1}{f})$$

Velocity of cart is —

$$\text{velocity} = \text{distance/time}$$

$$v = \frac{d}{t} = \frac{d}{n \times \frac{1}{f}} = \frac{df}{n}.$$

Notice that n is the number of time intervals in distance d, not the number of sparks. The paper tape will resemble that shown in Figure 2. Care should be taken to use only portions of the tape where the sparks are spaced evenly apart; that is, where the velocities are constant. Measure distances d_1, d_1', and d_2' on tape in meters, and record in Data Table 1. Count number of time intervals, n_1, n_1', n_2' corresponding to each distance, and record these in Data Table 2 also. The frequency f of spark timer has been determined by manufacturer and should be listed in equipment manual for the timer. Record this value in Data Table 1. Calculate the velocities v_1, v_1', and v_2' from the measured values d_1, d_1', d_2' and n_1, n_1', n_2' and frequency f. Record these in the "results" section of Data Table 1.

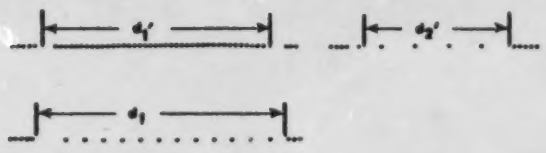


FIG. 2

5. Verify conservation of momentum by calculating and comparing the following:

   Momentum before collision = $m_1 v_1$

   Momentum after collision = $m_1 v_1' + m_2 v_2'$

   Record these values.

6. Secure slotted masses totaling approximately 100 g to the target cart with tape. Repeat steps 2, 3, and 4. Record results in Data Table 2.

INELASTIC COLLISION:

7. Remove one spring bumper from each cart, and stick putty on rear of target so that collision will be inelastic. Also remove timer wire from target. Repeat steps 2 and 3, but note that there will be only two spark tracks now rather than three. Verify conservation of momentum by calculating and comparing the following:

   Momentum before collision = $m_1 v_1$

   Momentum after collision = $(m_1 + m_2)v$

   Record these values in Data Table 3.

8. How might better automobile designs be produced by a thorough application of the principle of inelastic collision?

DATA TABLE 1: COLLISION BETWEEN EQUAL MASSES		
DATA		
m_1 = _____ kg	m_2 = _____ kg	f = _____/s
d_1 = _____ m	n_1 = _____	
d_1' = _____ m	n_1' = _____	
d_2' = _____ m	n_2' = _____	
RESULTS		
v_1 = _____ m/s	v_1' = _____ m/s	v_2' = _____ m/s
Momentum before collision = $m_1 v_1$ = _____ kg·m/s		
Momentum after collision = $m_1 v_1 + m_2 v_2$ = _____ kg·m/s		

DATA TABLE 2: COLLISION BETWEEN UNEQUAL MASSES
DATA
m_1 = _____ kg m_2 = _____ kg f = _____/s
d_1 = _____ m n_1 = _____
d_1' = _____ m n_1' = _____
d_2' = _____ m n_2' = _____
RESULTS
v_1 = _____ m/s v_1' = _____ m/s v_2' = _____ m/s
Momentum before collision = m_1v_1 = _____ kg·m/s
Momentum after collision = $m_1v_1' + m_2v_2'$ = _____ kg·m/s.

DATA TABLE 3: INELASTIC COLLISION
DATA
m_1 = _____ kg m_2 = _____ kg f = _____/s
d_1 = _____ m n_1 = _____
d' = _____ m n' = _____
RESULTS
v_1 = _____ m/s v = _____ m/s
Momentum before collision = m_1v_1 = _____ kg·m/s
Momentum after collision = $(m_1 + m_2)v$ = _____ kg·m/s.

SELF CHECK

1. State the principle of conservation of momentum (a) mathematically, identifying all symbols, and (b) verbally.

2. Define and give two examples of each of the following:

   a. Elastic collision.
   b. Inelastic collision.

3. A streetcar weighs 9072 kg empty and 9980 kg loaded with passengers. A loaded streetcar coasting at 12.0 m/s rams an empty streetcar initially at rest on the track. Find the final velocity of both streetcars if the collision is completely inelastic.

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Sears, Francis W.; Zemansky, Mark W.; and Young, Hugh D. University Physics. 5th ed. Reading, MA: Addison-Wesley Publishing Company, 1976.

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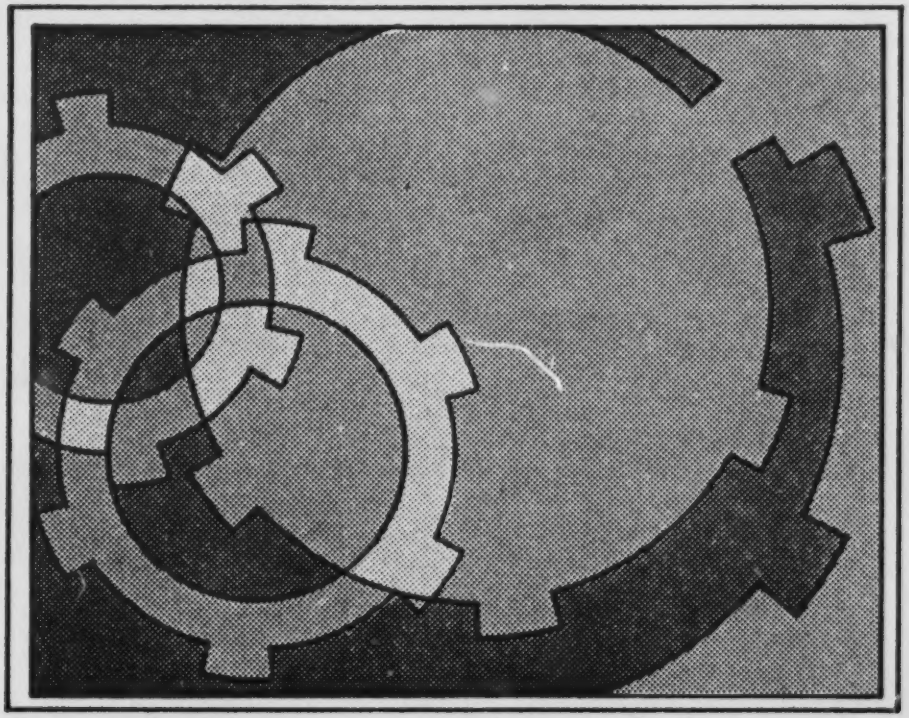
UNIFIED TECHNICAL CONCEPTS

MODULE 4 M 4

GYROSCOPES



MECHANICAL APPLICATION OF MOMENTUM

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

A gyroscope is a mechanical device that consists of a spinning wheel, called a "rotor," mounted on an axle. The ends of the axle are supported in a mounting stand. The unique properties of this device are based upon the "gyroscopic effect." The gyroscopic effect can be observed in such simple motions as riding a bicycle or spinning a top, but also provides the operating principles for complex navigational aids.

In this module, the student will learn how the properties of the gyroscope depend upon torque and angular momentum. The student will solve problems involving these parameters and gain an understanding of the vector nature of the gyroscopic effect.

In the laboratory, the student will observe the effects of external forces on a spinning gyroscope. The student also will use the gyroscopic effect to determine the angular momentum of a spinning gyroscope. Finally, the student will construct a simple gyrocompass.

PREREQUISITES

Student should have completed Module 4-0, "Momentum"; and Application Modules IM3, "Addition of Torques," and 4M1, "Angular Momentum of a Flywheel."

OBJECTIVES

Upon satisfactory completion of this module, the student should be able to:

- State the right-hand rule for determining the direction of a rotational vector.
- Label a diagram of a spinning gyroscope with the following vectors:

  Angular velocity, $\vec{\omega}$.
  Angular momentum, $\vec{L}$.
  Force, $\vec{F}$, acting on the gyroscope.

Torque, $\vec{\tau}$, acting on the gyroscope.
Angular momentum change, $\Delta\vec{L}$.
Angular velocity of precession, $\vec{\Omega}$.

- Given the magnitudes of two of the following vectors, find the magnitude of the third:

  Angular momentum, $\vec{L}$.
  Torque, $\vec{\tau}$.
  Angular velocity of precession, $\vec{\Omega}$.

- List the major parts of a gyrocompass, and explain the function of each.

- List the two methods for stabilizing a gyrocompass. Explain the effect of each on the magnitude of the torque $\vec{\tau}$, or the angular momentum $\vec{L}$.

- Find the direction of the gyroscopic effect on a moving vehicle.

- Given the appropriate equipment, demonstrate that the spin axis of a gyroscope tends to retain a fixed direction in space (in the absence of any torque on the gyroscope).

- Given the appropriate equipment, demonstrate that the direction of precession of a gyroscope is always perpendicular to the applied force.

- Given the necessary equipment, measure the precession rate of a spinning gyroscope and the torque acting upon it. Use these values to calculate the angular momentum of the gyroscope.

DISCUSSION

Before the gyroscopic effect can be discussed, a framework within which the pertinent rotational quantities can be treated as vectors must be developed. This requires a system for determining both the magnitude and the direction of such vector quantities as torque, angular velocity, and angular momentum.

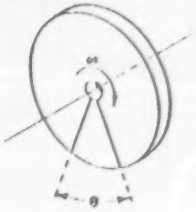


FIG. 1

Equations for determining the magnitudes of these quantities have been presented in previous modules. The magnitude of the angular velocity of the flywheel of Figure 1, for instance, can be calculated from Equation 1.

$$\omega = \frac{\theta}{t} \qquad (1)$$

where

- ω = average angular velocity in rad/s.
- θ = angle through which rotating body turns in time t; measured in rad/s.
- t = time during which rotating body advances through angle θ.

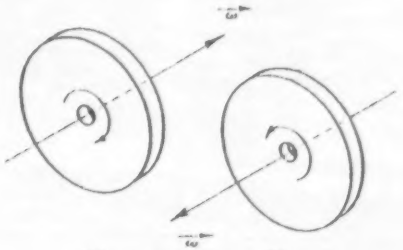


FIG. 2

The vector nature of angular velocity is evident from the fact that the flywheel can rotate in either of two directions — clockwise or counterclockwise. The vector representing $\vec{\omega}$ must then point in one direction to signify clockwise rotation and a different direction to signify counterclockwise rotation. The standard means of representing this vector is illustrated in Figure 2. The angular velocity vector of the clockwise-rotating flywheel lies along the axis of rotation pointing away from the viewer (Figure 2a). The angular-velocity vector of the counter-clockwise-rotating flywheel also lies along the axis of rotation, but points toward the viewer (Figure 2b).

In general, the vector associated with any rotational quantity lies along the axis of rotation in a direction determined by the right-hand rule. Wrap the fingers of the right hand around the axis of rotation in the direction of rotation. The extended thumb points in the direction of the rotational vector. Figure 3 indicates how to find the direction of $\vec{\omega}$ for the rotating flywheel using the right-hand rule.

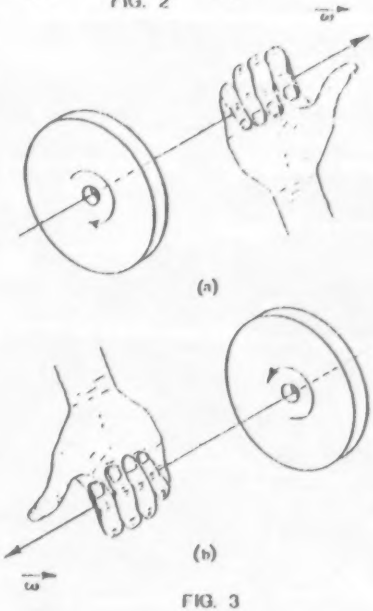


FIG. 3

This is a convenient point at which to introduce two new symbols to represent vectors aligned perpendicular to the plane of the page. The symbol ⊗ represents a vector pointing straight up

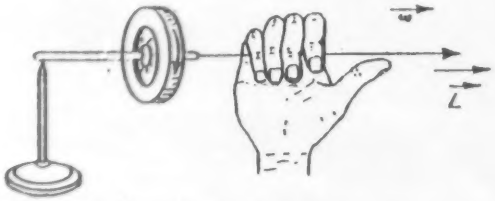
out of the page toward the reader. The symbol ⊗ represents a vector pointing straight down into the page, away from the reader.

Example A shows how the basic vector $\vec{\omega}$, $\vec{L}$, and $\vec{\tau}$ can be found for a spinning gyroscope supported at one end.

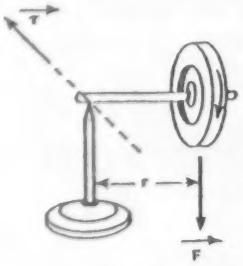
EXAMPLE A: ANGULAR VELOCITY, ANGULAR MOMENTUM AND TORQUE ON A SPINNING GYROSCOPE

Given: A gyroscope of mass 100g, moment of inertia 900g cm^2, and angular speed 3000 r/min (rpm) is supported as shown. The distance from the end of the gyroscope to its center of gravity is 4.50 cm.

Find: (a) its angular velocity $\vec{\omega}$ in rad/s.
(b) its angular momentum $\vec{L}$.



(c) The torque $\vec{\tau}$ acting upon it.



Solution: (a) To find the magnitude of $\vec{\omega}$ convert the units from rpm to rad/s.

$$\omega = 3000 \text{ rpm}$$

$$= \left(3000 \frac{r}{\text{min}}\right)\left(2\pi \frac{\text{rad}}{r}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$= \frac{6000\pi \text{ rad}}{60 \text{ s}}$$

$\omega = 314$ rad/s

The direction of $\vec{\omega}$ is found by wrapping the fingers of the right hand around the spin axis in the direction of rotation, as shown. The angular velocity vector now can be represented as follows:

$\xrightarrow{\vec{\omega}}$, where $|\omega| = 314$ rad/s

(b) Magnitude of $\vec{L}$:

angular momentum = moment of inertia x angular velocity,

$$L = I\omega \qquad \text{(Equation 4)}$$

$$= (900 \text{ g cm}^2)(314 \text{ rad/s})$$

$$L = 2.83 \times 10^5 \frac{\text{g cm}^2}{\text{s}}$$

Direction of $\vec{L}$:

The equation $L = I\omega$ is a vector equation, and indicates that the vector $\vec{L}$ equals the scalar I times the vector $\vec{\omega}$. Thus, L and ω have the same direction. The vectors $\vec{\omega}$ and $\vec{L}$ are shown below:

$\xrightarrow{\vec{\omega}}$ $\xrightarrow{\vec{L}}$ $\qquad |L| = (2.83)(10^5 \frac{\text{g cm}^2}{\text{s}})$

(c) Magnitude of τ:

torque = force x lever arm,

$$\tau = Fr$$

The force on the gyroscope is its weight, so

force = mass x acceleration of gravity,

$$F = mg$$

$$= (100 \text{ g})(980 \frac{\text{cm}}{\text{s}^2})$$

$$F = 9.80 \times 10^4 \text{ dynes}$$

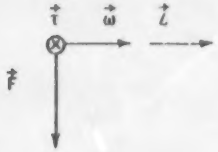
Then:

$$\tau = (9.80 \times 10^4 \text{ dynes})(4.50 \text{ cm})$$

$$\tau = 4.41 \times 10^5 \text{ dyne cm}$$

Direction of $\vec{\tau}$:

The force $\vec{F}$ acting upon the moment arm r, as shown in the figure above, tends to cause a rotation of the gyroscope assembly in a clockwise direction — as viewed by the reader. The product of F and r is equal to the torque. Thus, when the right-hand rule is applied to the figure shown above, the torque vector τ has the direction along the dashed line, as shown, away from the viewer. The vectors $\vec{\omega}$, $\vec{\tau}$, $\vec{L}$, and F are shown below:



PRECESSION

One of the most fascinating aspects of the gyroscope effect is that the gyroscope does not fall off the support. Instead, the free, unsupported end of the gyroscope moves in a horizontal circle with the support at its center. This motion is called "precession." The rate at which the spin axis precesses around the support is called the "angular velocity of precession," $\vec{\Omega}$.

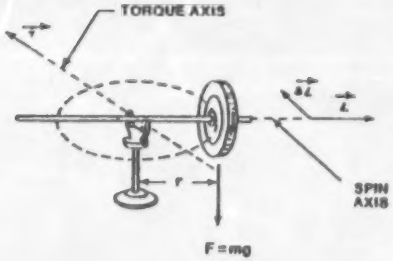


FIG. 4

Figure 4 shows a gyroscope precessing around its supported end.

To understand precession, one may begin with the relation between the angular impulse $\tau\Delta t$ and the angular momentum change $\Delta\vec{L}$. This already has been presented as Equation 7 in Concept Module 4-0:

$$\Delta L = \tau \Delta t \qquad (2)$$

where

ΔL = change in angular momentum (angular impulse).
τ = torque.
Δt = elapsed time.

Since $\vec{\tau}$ and $\Delta\vec{L}$ are vectors, the corresponding vector relationship is given by Equation 3.

$$\Delta\vec{L} = \vec{\tau}\Delta t \qquad (3)$$

Equation 3 states that the effect of a torque $\vec{\tau}$ acting upon the gyroscope for a length of time equal to Δt is to change the angular momentum of the gyroscope by an amount $\Delta\vec{L}$. Furthermore, the angular momentum changes in the direction of the applied torque. Example B shows how $\Delta\vec{L}$ is calculated for the gyroscope of Figure 4.

EXAMPLE B: $\overrightarrow{\Delta L}$ FOR A GYROSCOPE

Given: The gyroscope of Figure 4 has a mass m of 0.2 kg and a lever arm r of 0.05 m.

Find: Its angular momentum change $\Delta\vec{L}$ after an elapsed time of 0.03 seconds.

Solution: (a) Calculate the magnitude of the torque acting:

$$\tau = Fr$$

$$= mgr$$

$$= (0.2\ \text{kg})(9.8\ \text{m/s}^2)(0.05\ \text{m})$$

$$\tau = 0.098\ \frac{\text{kg m}^2}{\text{s}^2}$$

The direction of $\vec{\tau}$ is found by applying the right-hand rule around the torque axis.

(b) Find the magnitude of $\Delta\vec{L}$:

$$\Delta L = \tau\Delta t$$

$$= (0.098\ \frac{\text{kg m}^2}{\text{s}^2})(0.03\ \text{s})$$

$$\Delta L = 2.94 \times 10^{-3}\ \frac{\text{kg m}^2}{\text{s}}$$

From Equation 2, the directions of $\vec{\tau}$ and $\Delta\vec{L}$ are the same.

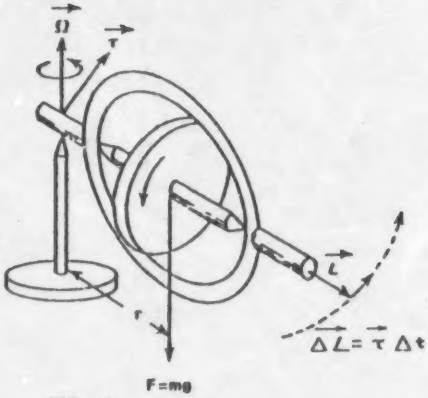


FIG. 5

Figure 5 shows a spinning gyroscope with its weight, torque, angular momentum, and angular momentum change vectors shown. The angular momentum change $\Delta\vec{L}$ after a short time Δt is given by Equation 3. The angular momentum $\vec{L}$ at the end of this time period must be the vector sum of the initial angular momentum $\vec{L}_0$ and $\Delta\vec{L}$:

$$\vec{L} = \vec{L}_0 + \Delta\vec{L}$$

Figure 6 shows these vectors added graphically. The final angular momentum $\vec{L}$ has approximately the same magnitude as $\vec{L}_0$, but is displaced through angle $\Delta\theta$ from $\vec{L}_0$. Since ΔL is perpendicular to L_0, the vector L_0 rotates but does not change in magnitude. Since $\vec{L}$ always lies along the spin axis, the spin axis rotates, or precesses, in the direction of $\Delta\vec{L}$. Notice that the torque vector rotates with $\Delta\vec{L}$, so that they remain parallel.

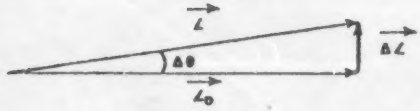


FIG. 6

Figure 6 can be used to estimate the magnitude of the angular velocity of precession $\vec{\Omega}$, which is just the rate of change of angle $\Delta\theta$. From Figure 6 –

$$\tan(\Delta\theta) = \frac{\Delta L}{L}$$

Since $\Delta\theta$ is a very small angle,

$$\tan(\Delta\theta) \approx \Delta\theta \text{ (in radians)}$$

Then –

$$\Delta\theta = \frac{\Delta L}{L}$$

$$\Delta\theta = \frac{\tau\Delta t}{L} \text{ (since } \Delta L = \tau\Delta t)$$

$$\frac{\Delta\theta}{\Delta t} = \frac{\tau}{L} \text{ (Both sides are divided by } \Delta t.)$$

$$\Omega = \frac{\tau}{L} \text{ (since } \Omega = \frac{\Delta\theta}{\Delta t})$$

Example C shows how to find the magnitude and direction of $\vec{\Omega}$ for a precessing gyroscope.

EXAMPLE C: ANGULAR VELOCITY OF PRECESSION FOR A GYROSCOPE

Given: The gyroscope of Example A ($m = 100$ g, $I = 900$ g cm^2, $r = 4.50$ cm) spins clockwise at the rate of 3000 rpm.

Find: Its angular velocity of precession $\vec{\Omega}$.

Solution: (a) Find $\vec{\tau}$:

$$\tau = Fr$$

$$= mgr \quad \text{(since } F = mg\text{, the gyroscope's weight)}$$

$$= (100\ \text{g})(980\ \text{cm/s}^2)(4.50\ \text{cm})$$

$$\tau = 4.41 \times 10^5\ \frac{\text{g cm}^2}{\text{s}^2}$$

The direction of τ is found by applying the right-hand rule around the torque axis.

(b) Find $\vec{L}$:

$$L = I\omega \quad \text{(Equation 4)}$$

$$= (900\ \text{g cm}^2)(3000\ \text{rpm})$$

$$= (900\ \text{g cm}^2)(3000\ \frac{\text{rev}}{\text{min}})(2\pi\ \frac{\text{rad}}{\text{r}})(\frac{1\ \text{min}}{60\ \text{s}})$$

$$L = 2.83 \times 10^5\ \frac{\text{g cm}^2}{\text{s}}$$

The direction of $\vec{L}$ is found by applying the right-hand rule around the spin axis.

(c) Calculate Ω:

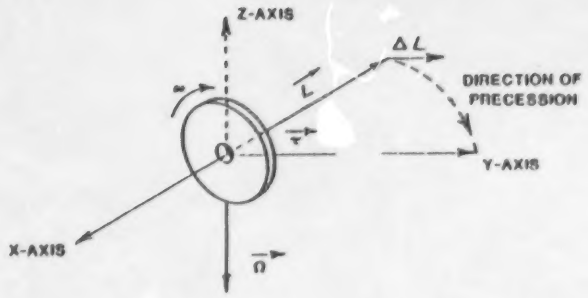
$$\Omega = \frac{\tau}{L}$$

$$= \frac{4.41 \times 10^5\ \frac{\text{g cm}^2}{\text{s}^2}}{2.83 \times 10^5\ \frac{\text{g cm}^2}{\text{s}}}$$

$$= (1.56\ \frac{\text{g cm}^2}{\text{s}^2})(\frac{\text{s}}{\text{g cm}^2})$$

$$= 1.56\ \frac{1}{\text{s}}$$

$$\Omega = 1.56\ \frac{\text{rad}}{\text{s}}$$



The direction of precession at each instant is the same as the direction of $\Delta\vec{L}$. Equation 3 indicates that $\Delta\vec{L}$ and $\vec{\tau}$ point in the same direction; so the gyroscope must precess in the direction of $\vec{\tau}$. Application of the right-hand rule around the precession axis indicates that the vector $\vec{\Omega}$ points down. (The gyroscope precesses along the dotted arc shown in the previous figure.)

(a)

(b)

FIG. 7

GYROSCOPIC STABILITY

Examination of Equation 4 shows that as the torque on the gyroscope becomes smaller, Ω also decreases. If the torque is reduced to zero in a model such as shown in Figure 8, the gyroscope stops precessing; and the spin axis remains rigidly fixed in space. This aspect of the gyroscopic effect is called "gyroscopic stability."

A bicycle or motorcycle being ridden demonstrates gyroscopic stability and precession. The spinning wheels of the cycle are gyroscopes. With the rider's weight positioned in the same vertical plane as the wheels (as in Figure 7a), there is no torque exerted on the wheels. The precessional velocity is zero, and gyroscopic stability keeps the bicycle upright. This is why the act of riding a bicycle is a very simple task, but balancing on a stationary cycle is very difficult. When the rider leans to either side, his or her weight exerts a torque that acts upon the gyroscopes (wheels). The wheels precess, and the bicycle turns right or left (Figure 7b). When we learn to ride a bicycle, we do this without thinking.

Gyroscopic stability allows the use of the gyroscope as a direction reference. A gyroscope mounted in such a manner that there is no torque acting upon it maintains its spin axis in the same direction indefinitely. This phenomenon is the basic operating principle of the gyrocompass diagrammed in Figure 8.

The gyroscope is mounted inside pivoted rings called "gimbals," which allow the base to be moved in any direction without exerting a torque upon the gyroscope. The inner gimbal allows the base to be turned left or right without exerting a torque upon the gyroscope. The outer gimbal allows the base to be tilted forward or backward without exerting a torque on the gyroscope. The gyroscope and gimbals are pivoted on jeweled bearings to reduce friction.

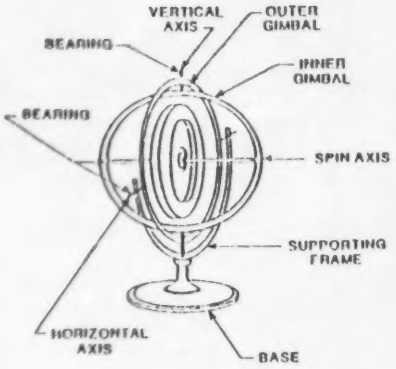


FIG. 8

In practice, no matter how finely the bearings are ground, they cannot be completely frictionless. Friction causes a small torque to be exerted on the gyroscope whenever the base is moved, resulting in precession, which rotates the spin axis out of alignment. This effect is minimized by making the amount of inertia of the rotor as large as possible and by spinning it at a very high angular velocity. Each of these steps serves to increase the angular momentum of the gyroscope. According to Equation 4, an increase in L causes Ω to decrease, making the precession less severe.

The gyrocompass is thus stabilized in two ways:

1. Gimbal mounting makes τ in Equation 4 very small.
2. Large values of I and ω make L in Equation 4 very large.

The gyrocompass used on most ships has a rotor 11 inches in diameter and 55 lbs in weight. It is spun by an electric motor at about 6000 rpm.

GYROSCOPIC EFFECT

In addition to the navigational uses of the gyrocompass, the gyroscopic direction finder has many applications. It is used in road mapping to measure accurately the curves in highways. It can measure curves and grades of sections of tract in railroad surveying. Gyroscopes also are attached to oil-drilling equipment to determine and to control the direction of drilling.

The gyroscopic effect often intrudes detrimentally on the operation of commonplace devices. The rotating flywheel of an engine is a gyroscope with a very large moment of inertia. The

effects of this large gyroscope are seen in the flight characteristics of single-engine aircraft. Figure 9a shows an airplane in level flight. The propeller and engine flywheel rotate clockwise (as viewed from the pilot's seat). The angular momentum $\vec{L}$ is found by application of the right-hand rule around the spin axis. Figure 9b shows the vectors associated with the aircraft as it goes into a dive. The force $\vec{F}$ pulls the nose down. The torque $\vec{\tau}$ is found by applying the right-hand rule around the torque axis, which causes precession around the precession axis in the direction of $\Delta\vec{L}$. Thus, the aircraft veers left when the nose is dropped.

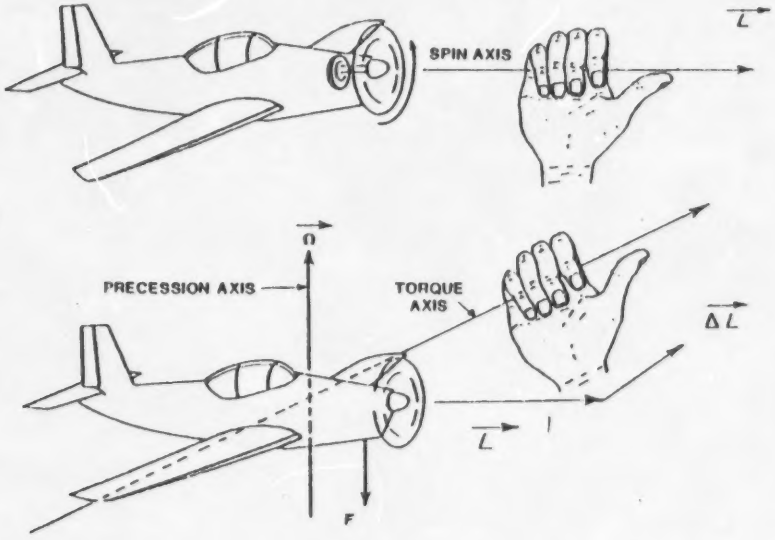


FIG. 9

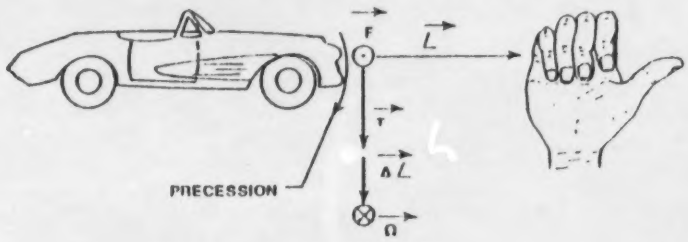
This effect is most noticeable when the angular momentum of the engine is small, as readily seen from Equation 4, since L appears in the denominator. The moment of inertia of the flywheel is constant, so the angular momentum can be changed only by changing the angular velocity of the engine. Therefore, the effect is most pronounced at low engine speeds (small angular momentum).

In twin-engine aircraft, the gyroscopic effect is nullified if each engine is designed to rotate in opposite directions. If the left engine rotates clockwise, the right engine rotates counterclockwise. This difference in direction causes the precessional motion of the engines to be equal in magnitude and opposite in direction, so that there is no net precession.

The gyroscopic effect of an engine in operation is illustrated in Example D.

EXAMPLE D: THE CORNERING AUTOMOBILE

Given: An automobile engine (and flywheel) rotates clockwise (viewed from the driver's seat). The auto makes a sharp right turn.



Find: The gyroscopic effect acting on the automobile.

Solution: Use the right-hand rule to find the direction of the angular momentum $\vec{L}$. The force on the auto is to the right, causing downward torque. The angular momentum change $\Delta\vec{L}$ is also downward. Since the precession is in the same direction as $\Delta\vec{L}$, the front end of the auto drops as the car goes around the corner.

SUMMARY

The gyroscope is a wheel and axle mounted in gimbals in such a manner that it is free to turn in any direction. When the wheel is caused to turn rapidly, it remains in a fixed position (by conservation of angular momentum), even if the base is moved. If a force is applied, however, at right angles to the axis of rotation, "precession" occurs. Precession is the movement of the axis at right angles to the rotation vector and to the torque applied to the axis. The gyroscopic effect is exhibited by bodies rotating at sufficient speed; these bodies include engine crankshafts. This effect is employed in such instruments as gyrocompasses (which indicate true north), direction indicators, automatic pilots, horizon indicators, and turn-and-bank indicators.

THE GYROSCOPIC EFFECT

MATERIALS

Demonstration gyroscope.
Unsharpened pencil.
String.
Stand or mount from which gyroscope can be hung.

PROCEDURE

1. Set the gyroscope spinning, and hang it with its spin axis horizontal. (See Figure 10.) Record in Data Table 1 the direction of spin (clockwise or counterclockwise as viewed along the spin axis from the front end of the spin axis).

2. Push near or front end of spin axis to left with pencil. Record direction in which gyroscope tends to precess. Repeat with pushes to the right, up and down.

3. Verify observed precessions for each of the above cases by drawing vector diagrams. Each diagram should include $\vec{L}$, $\vec{P}$, $\vec{\tau}$, $\vec{\Delta}$, and $\vec{\Omega}$.

4. Explain "precession."

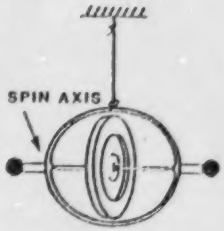


FIG. 10

PRECESSION RATE OF A GYROSCOPE

MATERIALS

Demonstration gyroscope and support.
Needle marker.
Meter stick.
Timer with accuracy of 0.1 seconds or better.
Balance with accuracy of 1 g or better.

PROCEDURE

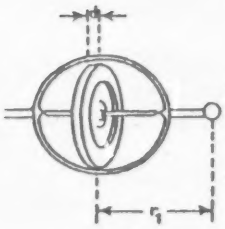


FIG. 11

1. Measure and record in Data Table 2 the distances r_1 (from one end of gyroscope to edge of rotor) and d (the thickness of rotor). (See Figure 11.)
2. Use balance to measure mass of gyroscope. Record this value.
3. Start gyroscope spinning. Record direction of spin.
4. Place one end of gyroscope on support, and allow it to begin precessing. In order to start gyroscope off with pure precession, push outer end gently in direction of precession.
5. Allow gyroscope to precess until spin axis is horizontal. Place a needle marker next to stand. (See Figure 12.) When axis of gyroscope precesses past marker, start timer. Stop timer when gyroscope next passes over needle marker. Record elapsed time.

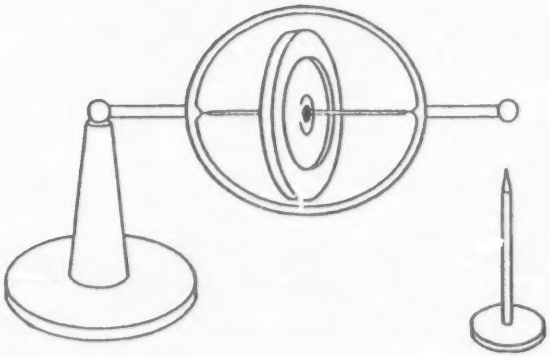


FIG. 12

6. Draw a vector diagram to verify the direction of precession.
7. Calculate weight F of gyroscope.
8. Calculate lever arm r of gyroscope.
9. Calculate magnitude of torque τ.
10. Calculate angular velocity of precession Ω.
11. Calculate angular momentum L of gyroscope.

GYROSCOPIC DIRECTION REFERENCE

MATERIALS

Demonstration gyroscope.
Two needle markers.
String.
Stand or mount on which gyroscope can be hung.

PROCEDURE

1. Hang spinning gyroscope with its spin axis horizontal, as illustrated in Figure 13. Mark the position of each end of axis with a needle marker.

2. Unhook string from mounting stand, and walk around room holding string from which gyroscope is suspended.

3. Return to mounting stand, and rehang gyroscope. The ends of spin axis still should be aligned with needle markers.

4. Why do the ends of the spin axis remain aligned?

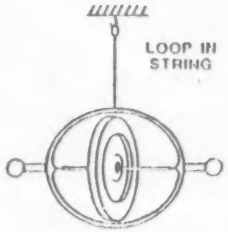


FIG. 13

DATA TABLE 1: THE GYROSCOPIC EFFECT	
Direction of spin: _____.	
Direction of push	Direction of precession
Left	_____
Right	_____
Up	_____
Down	_____

DATA TABLE 2: PRECESSION RATE OF A GYROSCOPE

Data:

r_1 = _____ cm d = _____ cm

Mass = m = _____ g

Direction of spin: _____

Elapsed time = t = _____ s

Angular displacement, $\theta = 2\pi$ rad

Calculations:

$F = (m)(980 \frac{cm}{s^2}) =$ _____ $\frac{g\ cm}{s^2}$

$r = r_1 + (1/2)d =$ _____ cm

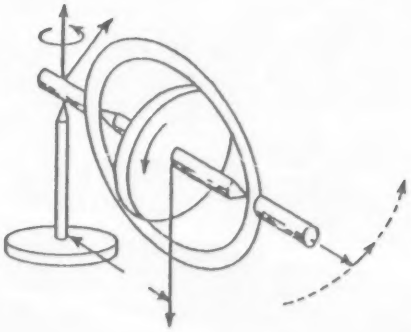
$\tau = Fr =$ _____

$\Omega = \theta/t =$ _____ rad/s

$I = \tau/\Omega =$ _____ $\frac{g\ cm^2}{s}$

SELF CHECK

1. State the right-hand rule for determining the direction of a rotational vector.



2. The rotor of a gyroscope spins as shown at left. Label the diagram with the following vectors:

   Angular velocity, $\vec{\omega}$.
   Angular momentum, $\vec{L}$.
   Force (weight), $\vec{F}$.
   Torque, $\vec{\tau}$.
   Angular momentum change, $\Delta\vec{L}$.
   Angular velocity of precession, $\vec{\Omega}$.

3. A spinning gyroscope precesses at the rate of 0.3 rad/s and has angular momentum 5000 rpm. Find the torque acting upon the gyroscope.

4. List the major parts of a gyrocompass, and explain the function of each.

5. List the two methods of stabilizing a gyrocompass. Explain the effect of each upon the magnitude of $\vec{\tau}$ or $\vec{L}$.

6. The aircraft of Figure 9a turns to the left. Find the direction of the gyroscopic effect on the aircraft.

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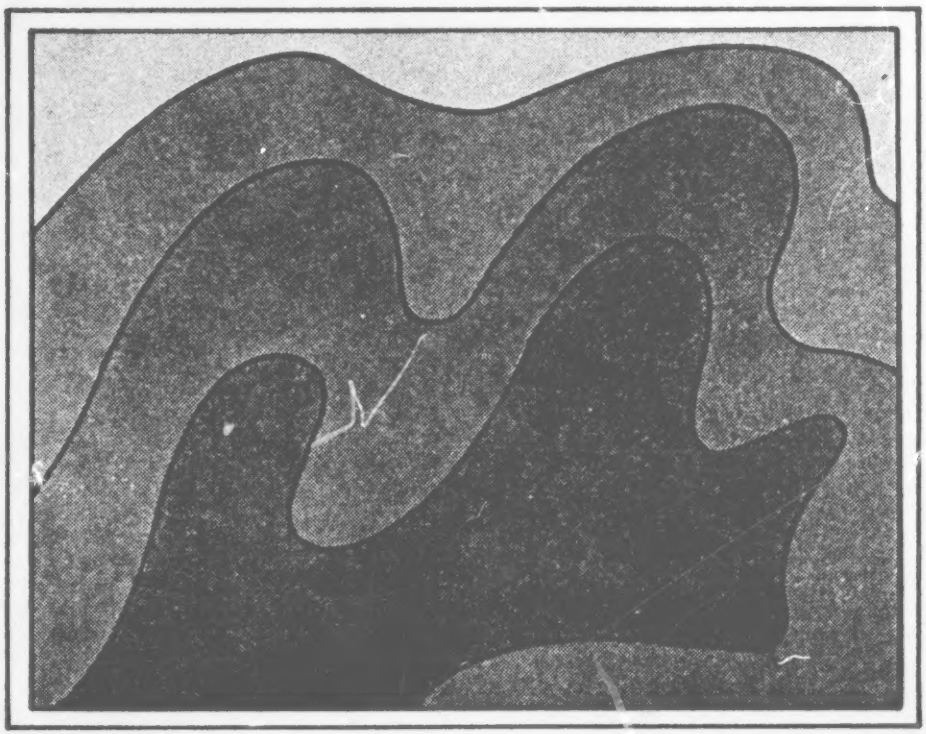
UNIFIED TECHNICAL CONCEPTS

MODULE 4 F 1

ROCKET ENGINES



FLUID APPLICATION OF MOMENTUM

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Various types of rocket engines available for study are solid fuel, liquid fuel (chemical propulsion), nuclear propulsion, electrical propulsion and solar propulsion. In this module, the solid fuel chemical propulsion rocket engine will be described as an application of the concept of momentum and impulse. This type of rocket engine is particularly representative of present-day military missile engines and is even available in model form for the propulsion of model rockets that can be purchased at toy stores.

In this module, general characteristics of rocket propulsion will be discussed. Concepts of momentum and impulse will be applied to the performance of a rocket engine. Characteristics affecting the impulse produced by solid fuel rocket engines are also discussed. An experiment is included to provide a hands-on test of the impulse produced by ignition of a solid fuel model rocket engine.

PREREQUISITES

Student should have completed Concept Module 4-0, "Momentum."

OBJECTIVES

To complete this module, the student should be able to:

- Briefly describe operation of a rocket engine.
- Calculate specific impulse of a rocket fuel, given the total impulse of the rocket engine and the weight of propellant burned.
- Estimate total impulse of a rocket engine given a thrust-time graph.
- Calculate burnout velocity of a model rocket, given the total impulse (change in momentum) of the rocket engine.

BASIC OPERATING PRINCIPLES

The rocket engine is a primary mover of military missiles, spacecraft, model rockets and 4th of July skyrockets. Figure 1 illustrates a basic simplified rocket engine. Fuel-burning rocket engines do not require an air supply to operate, as they usually carry an oxidizer that provides the oxygen to support combustion of the fuel. The fuel and oxidizer together are known as propellants.

Behind the propellant is the combustor, a space where the fuel is burned. A nozzle is located on the end of the engine through which the rapidly expelled combustion gases pass. The nozzle and combustion chamber are designed to control the rate at which combustion takes place. Thrusting force on the rocket is generated because the gas pressure - which would be equal in all directions in a closed space - is unbalanced by the open nozzle through which the gases can stream out. This leaves the pressure in the opposite direction (against the forward end) unopposed. It is this reactive force that drives the rocket forward, rather than a "push" against the outside atmosphere. Figure 2 shows the forces involved. Similar reaction forces make a pistol kick back when fired, or a cannon recoil against its mounting. A rocket uses this reactive force to propel itself. The same effect has been observed when a balloon filled with air is released and darts about until the air has been expelled.

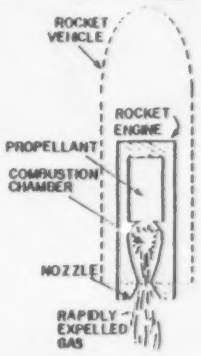


Fig. 1

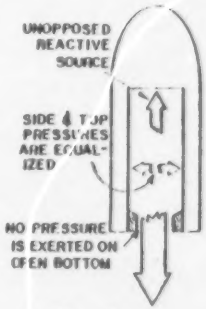


Fig. 2

CONSERVATION OF MOMENTUM APPLIED TO ROCKETS

Operation of rockets is an application of the law of conservation of momentum. During each instant of the time of flight, the total momentum of the rocket and the fuel must be constant. To illustrate this fact, consider the simplified rocket shown in Figure 3 at two separate instants of time. In the first instant of time, assume that the rocket has an instantaneous velocity V and a mass m, and contains an element of fuel Δm. The fuel element is ignited and leaves the rocket in the form of gases, now moving forward with a

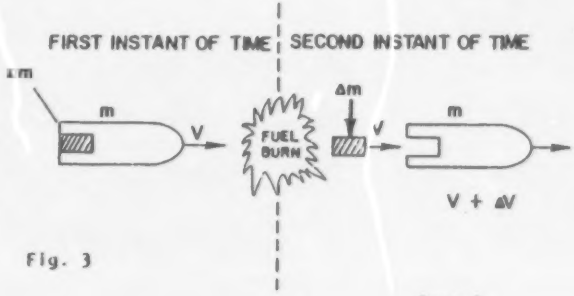


Fig. 3

smaller velocity v. In the second instant of time, the separation of the fuel element Δm from the rocket of mass m has occurred, producing an increase ΔV in velocity of the rocket.

$$\left(\begin{matrix}\text{Total momentum of}\\ \text{rocket + fuel}\end{matrix}\right) \text{ at instant 1} =$$

$$\left(\begin{matrix}\text{total momentum of}\\ \text{rocket + fuel}\end{matrix}\right) \text{ at instant 2}$$

$$[mV + (\Delta m)V]_{\text{instant 1}} = [(\Delta m)v + (V + \Delta V)]_{\text{instant 2}}. \qquad (1)$$

Simplifying Equation 1:

$$mV + \Delta mV = \Delta mv + mV + m\Delta V$$

Grouping the terms involving Δm on the left:

$$\Delta mV - \Delta mv = m\Delta V \qquad (2)$$

Factoring Δm:

$$-(v - V)\,\Delta m = m\Delta V \qquad (3)$$

Dividing both sides of Equation 3 by the time interval, Δt (Δt = instant 2 - instant 1):

$$-(v - V)\,\frac{\Delta m}{\Delta t} = m\,\left(\frac{\Delta V}{\Delta t}\right)$$

The quantity (v - V) is the velocity of the fuel gases relative to the rocket and is symbolized by c. Therefore, the previous equation becomes:

$$-c\,\left(\frac{\Delta m}{\Delta t}\right) = ma$$

where a is rocket acceleration. According to Newton's second law of motion, F = ma, the reactive force on the rocket is given by Equation 4.

$$-c\left(\frac{\Delta m}{\Delta t}\right) = F_{reactive} \qquad (4)$$

where

$F_{reactive}$ = thrusting force.

c = velocity of escaping fuel (a negative number).

$\frac{\Delta m}{\Delta t}$ = fuel mass flow rate.

Notice that c, as defined, equals (v - V). It is the velocity of the burned gases relative to the rocket. This will always be a velocity directed backward, hence it will be a negative quantity. Thus, in the above equation (-c) will be overall positive so that direction of the reactive force will always be positive - or forward - as it should be.

It is desirable in rocket design to have velocity c of the escaping mass as high as possible. Fuel consumption rate $\Delta m/\Delta t$ may be relatively small, however, because the rocket must carry all of its fuel.

Thrust varies during the duration of the burn of the rocket engine. Thrust is not used as an indicator of the rocket's overall performance. Rocket engineers use a quantity called specific impulse (I_{sp}), defined as the total impulse (I_t) obtained from the combustion of a unit weight (W_f) of propellant. This is summarized by Equation 5.

$$I_{sp} = \frac{I_t}{W_f} \qquad (5)$$

For example, a propellant composed of powdered zinc and sulphur would produce a specific impulse of 20 to 50 lb-s/lb, depending on the percentages of zinc and sulphur, and the engine design. The specific impulse of a rocket motor fuel can be calculated by measuring total impulse of the rocket engine and dividing by weight of the propellant burned.

An interesting and basic relationship exists between the specific impulse of a propellant fuel, I_{sp}, and the exhaust velocity of the fuel, c. Rearranging Equation 4:

$$c = \frac{-F\Delta t}{\Delta m}$$

Dividing both sides by g, the acceleration of gravity on earth:

$$\frac{c}{g} = \frac{-F\Delta t}{\Delta mg} = \frac{-F\Delta t}{\Delta W_f}$$

and, as the total impulse produced for the element of weight can be considered to be $(-F\Delta t)$, we find:

$$\frac{c}{g} = \frac{I_t}{\Delta W_f} = I_{sp}$$

or

$$c = gI_{sp} \qquad (6)$$

Thus, if specific impulse of a propellant is known, the exhaust velocity can be calculated by using Equation 6.

THRUST-TIME CURVES

Total impulse (change in momentum) produced by a rocket engine can be calculated by firing the engine and recording values of the thrust force produced throughout the duration of the burn. In Concept Module 5-0, "Momentum," it was pointed out that in "real-life" examples of impulse, magnitude of the force varies during the time the momentum changes are occurring. A typical force-time graph produced during the firing of a model solid fuel rocket engine is illustrated in Figure 4.

The particular shape of the thrust curve is a result of the design of the combustion area inside the rocket engine. For model rockets, it is highly desirable to have a large thrust early

In the burn, allowing the rocket to gain speed quickly so that it will have stability as it leaves the launch stand. For the rest of the burn, a constant thrust is desirable, as it will make possible a stable flight and a higher altitude gain (longer burn time).

The total impulse, as indicated in Concept Module 5-0, will equal the area under the thrust-time curve. Calculating the exact area under the curve is rather difficult with simple tools; however, if the curve is produced on graph paper or grid paper, the area under the curve can be estimated. In Figure 4, one square of the grid corresponds to an impulse-area of 0.2 lb x s. If the number of whole and fractional squares are added, the result can be multiplied by the 0.2 lb x s to obtain the estimate of the total impulse. In Figure 4, there are 15 whole squares + 5 squares (fractional parts added) = 20 squares (estimate). Can you calculate the estimated impulse?

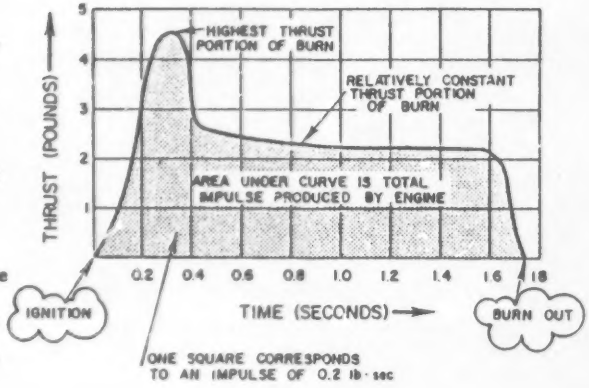


Fig. 4

Model rocketeers use total impulse of the model rocket engine and burnout weight of the model to accurately compute the burnout speed.

Total impulse is the total change in momentum produced in the rocket by the action of the engine, as shown in Equation 7.

$$I_{total} = (mv)_{burnout} - (mv)_{ignition} \qquad (7)$$

Because velocity is zero at ignition, the equation reduced to:

$$I_{total} = (mv)_{burnout}$$

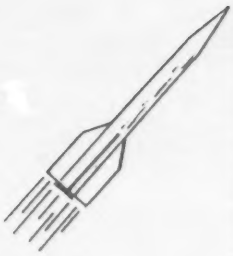
or

$$I_{total} = m_{burnout}\, v_{burnout}$$

where $m_{burnout}$ is the mass of the rocket after the fuel has been burned. Solving for $v_{burnout}$:

$$v_{burnout} = \frac{I_{total}}{m_{burnout}}. \qquad (8)$$

An example of the use of Equation 8 is given in Example A.



EXAMPLE A: CALCULATION OF BURNOUT VELOCITY

Given: Model rocket engine with a total impulse of 5 newton-seconds and a burnout mass of 40 grams.

Find: Burnout velocity of the rocket when its fuel is exhausted.

Solution:

$$v_{burn} = \frac{I_{total}}{m_{burnout}}$$

$$v_{burnout} = \frac{(5 \text{ newton-seconds})}{(40 \text{ grams} \times \frac{1 \text{ kilogram}}{1000 \text{ grams}})}$$

$$v_{burnout} = 125 \frac{m}{s}$$

MODEL ROCKET ENGINE STATIC FIRE TEST STAND

MATERIALS

Small d.c. motor (a small display motor will work) and motor mounting.
Variable d.c. power supply (0 to 12 volts).
Wooden base board (1 1/2 in x 5 in x 2 ft).
Central support wooden board (1 1/2 in x 5 in x 6 1/2 in).
Metal end-support plate (0.080-in aluminum plate, 5 in x 18 in with right angle bend 6 in from one end).
Two support rods for rocket engine sled, 1/4-in to 1/2-in diameter, 40 cm long threaded on end (Cenco Laboratory stand rods work nicely).
Two compression springs - chosen to slide on support rods, having approximately a 6-inch compression length for a maximum of 4 or 5 pounds compressive force (available from machine supply companies).
Two right angle metal support hardware 4-inch arms.
Four right angle metal support hardware 2-inch arms.
Wood screws for above hardware.
Recording drum made of heavy paperboard tube (a discard item from an IBM copier machine) 8 1/2 in long, 3 1/4 in diameter.
Wooden circle supports matching recording drum to its support rod.
Central support rod for recording drum.
Rubber tubing (coupling for d.c. motor to recording drum rod).
Rocket sled (11 cm x 16 cm approximately 0.037 thick aluminum) ends bent with 3 cm long right angle tabs with holes drilled such that sled slides freely on support rods.
Rocket engine mount (old metal 35 mm photographic film can) or equivalent size metal tube with one closed end.
Two small hold-down tension springs (available from machine supply companies) for easy mounting of rocket engines to sled.
Index rod - 0.6-cm diameter light aluminum rod threaded one end with nut.
Recording pen holder (laboratory thermometer clamp - modified to attach to index rod).
Supply of fresh felt-tip pens.
8 1/2 in x 11 in graph paper (10 x 10 to inch).
Microclips (for ignition circuit).
Lamp cord, 9 ft long (for ignition circuit).
Doorbell switch (for ignition circuit).
Twelve-volt automobile battery (for ignition circuit).

Model rocket engine. (Suggestion: Estes Industries D 12-0 booster stage model rocket engine #1665, with accompanying igniter wires - Estes Industries, Penrose, CO 81240.) Caution: Do not choose an engine that has a delayed parachute deployment charge in the front end of the engine.

PROCEDURE

1. Arrange model rocket engine test stand and associated equipment as shown in Figure 5. A cautionary note: Arrange this test so that apparatus is affixed to some heavy object; clear area of people for 10 ft around stand, 15 ft behind rocket engine before ignition.

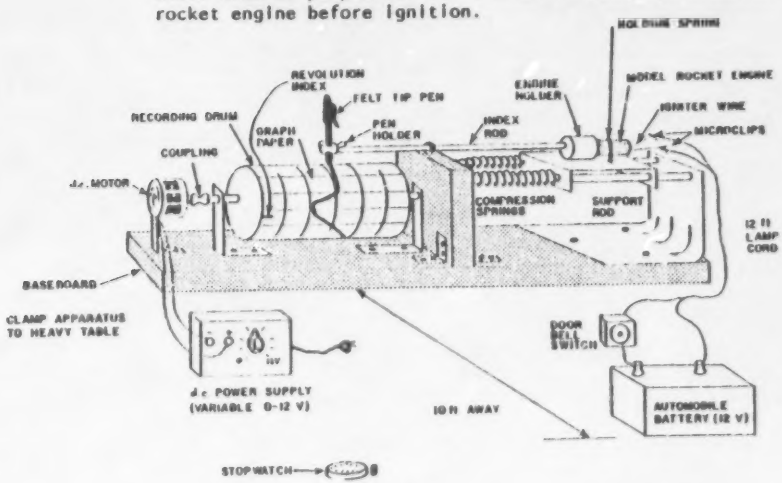


Fig. 5

In event of a misfire, check electrical igniter wires to make sure that igniter wires are in contact with fuel. Consult manufacturer's instructions for hints to improve ignition characteristics. The rocket engine suggested in this module is purchased with igniter wires and instructions.

2. Mount solid propellant model rocket engine in engine holder on test stand; make sure that it is secure and not loose in holder. Connect microclips to igniter wires embedded in engine nozzle.
3. Attach a sheet of 8 1/2 in x 11 in graph paper to recording drum with scotch tape.
4. Mark a broad dark line on graph paper for a revolution index as shown in Figure 5.
5. Adjust voltage to d.c. motor, to cause recording drum to begin to rotate slowly at a rate of one revolution in two seconds. This may be accomplished by counting approximately 30 revolutions, observing revolution index mark, for a time of one minute on stopwatch. (Thrust duration for suggested engine is approximately 1.8 seconds.)
6. Insert a fresh felt-tip pen in pen holder of apparatus; adjust pen height to produce a solid trace on graph paper.
7. Move all people away to firing location or beyond.
8. Fire rocket engine by depressing doorbell switch.
9. After engine has fired, reduce voltage to recording drum motor to stop drum.
10. Remove graph paper from drum; lay graph paper flat; identify coordinate axis (time and thrust force).
11. Compare curve with curve shown in manufacturer's instruction sheet.

SELF CHECK

1. Estimate the total impulse of the rocket engine whose thrust-time curve is shown in Figure 6.
   a. What is the impulse for one square in the graph?
   b. Count whole squares under the curve.
   c. Count fractional parts of the square under the curve; add these fractions.
   d. Calculate estimated total impulse by multiplying a by the sum of b and c. Total impulse (estimate) _____.
2. Calculate the specific impulse of a rocket whose engine produces a total impulse of 30,000 lb x s for 200 lb of propellant

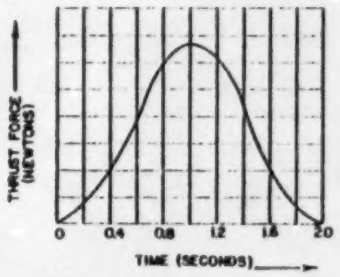


Fig. 6

burned.

3. A certain rocket has a weight of 6000 lb with no fuel and 12,000 lb with fuel. In flight, it ejects fuel at 250 lb/s with a relative exhaust velocity of 5000 ft/s with respect to the rocket. Calculate:
   a. Thrust of the rocket.
   b. Duration of the burn.
   c. Burnout velocity of this rocket in free space if it starts from rest.
4. The total impulse of a model rocket is 8 lb x s and the burnout mass of the rocket is 4 ounces. Calculate velocity of the rocket at burnout.
5. Describe the basic operation of a simple rocket engine including the functions of the propellant, combustion chamber and nozzle.

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